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Theory of Ionization by Combination Action and the Low Voltage Arc. KARL T. COMPTON 329

The Thermionic Work Function of Tungsten. C. DAVISSON AND L. H. GERMER 339

The Relative Sensitivity of the Eye at Different Levels of Luminance. DONALD MACKENZIE 351

The Optical Properties of Molten Metals. A. E. ASTIN 360

Bombardment of Metal Surfaces by Slow-Moving Electrons. H. E. FARNSWORTH 366

Note on the Specific Heat of Tungsten. PAUL F. GARDE 373

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THE

PHYSICAL REVIEW.

THEORY OF IONIZATION BY CUMULATIVE ACTION AND THE LOW VOLTAGE ARC.

By Karl T. Compton.

Synopsis.

Theory of Ionization by Cumulative Action of Successive Impacts by Electrons and by Quanta of Resonance Radiation.—The phenomena of the low-voltage arc seem to require, for their explanation, ionization of the gas molecules in two (or more) successive stages. In the theory here developed it is supposed that each electron from the cathode after falling through a minimum potential difference reaches an active zone in which it collides either with a neutral molecule and partially ionizes it or with a partially ionized molecule which thereby becomes completely ionized. Each molecule when partially ionized emits a quantum of resonance radiation and this may go off in any direction and will in a short distance be absorbed by a neutral molecule which thereby becomes partially ionized and emits another quantum of radiation. So the passage through the gas of the quanta of resonance radiation initially set free by electronic impacts is analogous to the diffusion of foreign gas molecules. *Equations for the case of coaxial cylindrical electrodes* are derived for the proportion of molecules partially ionized (1) by direct impact and (2) by resonance radiation in terms of quantities which can all be readily measured except τ the mean life of the ionized molecules and $1/\rho$ the scattering coefficient for the radiation, for which (except ρ for mercury) only upper and lower limits can be estimated. The substitution of experimentally determined values leads to the conclusion that, under normal circumstances, partial ionization by photo-impact is many times as great as that by electronic impact alone and is necessary and sufficient to account for the observed ionization. The possibility of complete ionization by successive photo-impacts alone is also discussed.

Theory of Low-voltage Arc.—If n_0 is the number of electrons emitted and P_0 is the proportion of partially ionized molecules in the active zone, n_0P_0 is the number of molecules ionized per second. Since the positive ions move more slowly than electrons, each positive ion neutralizes the space charge due to $4\sqrt{(2 \times 1840M)}$ electrons; hence the electronic current increases to $1/(1 - 242P_0\sqrt{M})$ times the value it would have without ionization, provided the current is limited by the space charge around the cathode. As P_0 is increased by increasing the temperature of the filament or the applied potential, the current first increases to the saturation thermionic current, then the negative space charge is replaced by a positive space charge, the potential drop becomes concentrated near the cathode, the temperature of the cathode is raised by bombardment by positive ions, increasing n_0 and hence P_0; as a result the current increases at an accelerated rate, instability is usually soon reached and the arc strikes suddenly. The chief function of the gas is to give the

positive space charge around the cathode which is the distinguishing feature of the arc. If n is the saturation value of thermionic emission, the maximum current will be $3/2ne$ or $2ne$ depending on whether the voltage is below or above the minimum ionizing potential.

Low-voltage Arc in Mercury Vapor.—Recent experiments indicate that the *striking voltage* is about 5.6 instead of 4.9 volts, and that the arc may be dependent on either the 4.9 volt (2536 Å.) or the 6.7 volt (1849 Å.) radiation according to the age of the vapor. Thus there seem to be *two meta-stable states of the neutral mercury atom.*

Theory of Temperature of Ionization of Gases.—It is pointed out that resonance radiation must also be the chief factor in temperature ionization both in the electric furnace and in the sun and other stars.

1. INTRODUCTION.

IN two recent papers the writer emphasized the impossibility of explaining low voltage arcs by ionization of gas molecules by single electron impacts.[1] By a low voltage arc, is meant one which strikes and operates at a voltage less than the minimum ionizing potential of the gas. The phenomena seem to demand, for their explanation, ionization of gas molecules in successive stages by two or more separate agents. Hughes has suggested the name "cumulative ionization" to describe such effects and has given a survey of the subject in his recent Report on Photoelectricity.[2] Cumulative action is probably an important factor also in temperature ionization, but it is in the low voltage arc that it can be most easily investigated.

There are three possible processes of cumulative ionization which seem reasonable in the light of our present knowledge: (1) Ionization by Successive Impact; (2) Photo-Impact Ionization; (3) Ionization by Successive Photoelectric Action. In the first of the writer's earlier papers an attempt was made to test the first of these possibilities. The equations there derived, however, are not accurately applicable to actual experimental conditions, so that the conclusions reached in the former paper must be considered as quite tentative. In the present paper the first two of these processes are tested by comparing the amount of ionization observed experimentally with the amount predicted by each of the processes under the conditions of the experiment. While there is uncertainty regarding the magnitude of several quantities appearing in the equations, yet the argument leads to some important and fairly definite conclusions.

2. (*a*) TOTAL CURRENT IN ABSENCE OF ARC.

Let Fig. 1 represent a discharge tube, in which A is the anode and C is the incandescent cathode which emits the electrons whose impacts

[1] PHYS. REV., 15, p. 476, 1920; Phil. Mag., 43, p. 531, 1922.

[2] National Research Council Bull., Vol. 3, No. 10.

against gas molecules may result in their complete or partial ionization. If the applied accelerating potential difference V exceeds the minimum radiating potential V_r of the gas, there will be inelastic collisions, and partial ionization of some of the molecules, together with emission of resonance radiation, will occur. Let the shaded area represent the region in which, on the average, these effective impacts occur. As a result of these collisions, suppose a fraction P of the molecules in this shaded region to be in a partially ionized state, capable of being completely ionized by the impact of any electron which has not yet lost its energy. If n electrons leave the cathode, then Pn is the resulting amount of ionization.

Each molecule thus ionized adds to the total current in two ways: (1) by contributing two additional ions, and (2) by the effect of the relatively slowly moving positive ions in neutralizing some of the space charge around the filament and thus permitting the escape of more electrons from it. These additional electrons may also be effective in producing further ionization, which will liberate more electrons, and so on. The total resulting current can thus be expressed as the sum of an infinite number of terms, forming a converging series, which can readily be expressed in a simple form as follows.

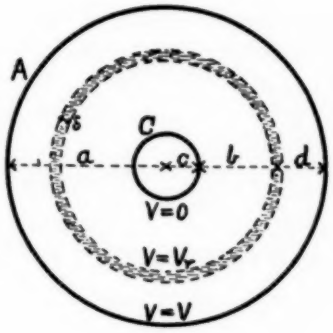

Fig. 1.

Each positive ion is drawn toward the cathode, through the outward stream of emitted electrons, and both make numerous collisions with molecules, since the phenomena in which we are interested are important only provided the mean free path is small in comparison with the distance between the electrodes. The average rate of motion of an ion through the gas is proportional to $\lambda\bar{c}$, where λ is its mean free path and $\bar{c}$ is its average velocity. The mean free path of the electrons is $4\sqrt{2}$ times greater than that of the ions and their average velocity is $\sqrt{1840M}$ times greater, where M is the molecular weight on the basis of 1 for the hydrogen atom. Thus each positive ion remains in the region of the negative space charge $4\sqrt{3680M}$ times as long as an escaping electron, neutralizes the space charge of that number of electrons and permits that number to escape from the cathode. Furthermore, as will be shown later, the probability P of partial ionization of a molecule in the shaded region is necessarily proportional to the number of bombarding electrons.

Let n_0 be the number of electrons which would leave the cathode if there were no ionization. Due to these, a fraction P_0 of the molecules in the shaded region are partially ionized and the resulting number of molecules completely ionized is n_0P_0. This number of positive ions liberates $4\sqrt{3680Mn_0P_0}$ additional electrons from the cathode. Thus

$$n' = n_0(1 + 4\sqrt{3680MP_0})$$

electrons would leave the cathode if only the original n_0 could ionize. Putting, for the present, $4\sqrt{3680MP_0} = Cn_0$, we have

$$n' = n_0(1 + Cn_0).$$

But, if we include the effect of ionization due to those electrons liberated by positive ions produced by the n_0 electrons, we have

$$n'' = n_0(1 + Cn') = n_0(1 + Cn_0 + C^2n_0^2).$$

Similarly

$$n''' = n_0(1 + Cn'') = n_0(1 + Cn_0 + C^2n_0^2 + C^3n_0^3)$$

is the number of emitted electrons if we include ionization due to all the n'' electrons. The actual total number of electrons leaving the cathode is seen to be

$$n = n_0(1 + Cn_0 + C^2n_0^2 + \cdots) = \frac{n_0}{1 - Cn_0}.$$

Thus we have

$$n = \frac{n_0}{1 - 4\sqrt{3680MP_0}}, \qquad (1)$$

provided the current is limited by the negative space charge around the cathode.

It is easily possible to measure n and to calculate n_0 by the relation[1] $n_0 \propto V^{3/2}$, determining the constant of proportionality by measurements of n at values of V less than V_r, where n and n_0 are identical. Thus equation (1) enables definite values of P to be determined under definite experimental conditions and permits a quantitative test of the values of P predicted by the theories of ionization by successive impact and photo-impact ionization to be outlined later.

Incidentally, this equation offers an explanation of the striking of an arc. As P_0 is increased, by increasing n_0 or the applied potential, n increases more and more rapidly and would become infinite when

$$P_0 = 1/4\sqrt{3680M}$$

[1] Langmuir, PHYS. REV., 2, p. 402, 1913.

were it not limited to the value corresponding to the saturation thermionic current at the temperature of the cathode. The number of available electrons being thus limited, the negative space charge becomes neutralized and is finally replaced by a positive space charge as the voltage is raised. As the space charge becomes positive, the potential drop becomes concentrated near the cathode, so that the electrons attain their critical speed within a short distance of the cathode. This concentration of the region of effective impacts tends further to increase the probability of cumulative ionization, measured by P. At the same time the temperature of the cathode is raised by the bombardment of the positive ions, which increases n_0 and P_0. Thus, after ionization sets in, the current increases at an accelerated rate as the voltage is raised. This increase may be slow or rapid, depending on the value of n_0, so that the setting in of the arc may be made gradual enough to be experimentally followed in all of its stages. Usually, however, the conditions near the arcing voltage are such that P_0 spontaneously increases as a result of the more favorable distribution of potential and the increase in the cathode temperature, so that a point of instability is reached at which the arc strikes suddenly,—the further increase of current occurring spontaneously. In any case the presence of a positive space charge around the cathode is the distinguishing feature of an arc.

2. (*b*) Total Current in Arc.

If the arc is operating at a voltage a little larger than the minimum ionizing potential V_i, the maximum current is obtained if each of the n electrons emitted from the cathode ionizes a molecule, so that the total current is $(n + n)e = 2ne$.

If the arc operates at a voltage between the minimum radiating potential V_r and the minimum ionizing potential V_i, the maximum possible current occurs when half of the n electrons collide to partially ionize molecules and the other half complete the process of ionization. Thus, in the low voltage arc, the maximum current is $(n + n/2)e = 3/2ne$. In both cases n is the saturation value of thermionic emission from the cathode at its temperature.

Recombinations would diminish the above values somewhat. Probably this decrease due to recombination is not large in the case of an intense arc, since each loss by recombination liberates radiant energy which may contribute to another ionization, and since the ionization occurs close to the cathode so that there is only a small region in which electrons and positive ions exist together.

Thus the chief function of the gas in a low voltage arc is to give a positive space charge around the cathode, causing sufficient potential

gradient to give a saturation current with low voltages, and to raise the cathode temperature by ionic bombardments and thus increase the saturation current.

The foregoing conclusions have received very satisfactory experimental support, which will be discussed in a later paper.

3. THEORY OF IONIZATION BY SUCCESSIVE IMPACTS.

The n electrons per second from the filament are drawn through the gas and attain a speed corresponding to the minimum radiating potential V_r. Each will lose its energy at some subsequent impact and partially or completely ionize the impacted molecule, according as the molecule was in the normal or partially ionized state before impact. Let τ be the average interval of time during which a molecule remains partially ionized, *i.e.*, the average interval between excitation and radiation. If δ is the thickness and f the length of the layer within which, on the average, these effective impacts occur, and if a fraction P of the molecules are partially ionized at any instant, then $n(1 - P)$ molecules are partially ionized each second. The aggregate time of activation of all molecules partially ionized in the layer δ per second is $n(1 - P)\tau$. This, divided by the number of molecules in the layer, gives the fraction of the molecules in the region of effective impacts which are, at any instant, in the partially ionized state as the result of a direct electron impact. Calling this P_i, we have

$$P_i = \frac{n(1 - P)\tau}{vNp},$$

where p is the gas pressure, N the number of gas molecules per unit volume at unit pressure and v the volume of the layer in which effective impacts occur. If the electrodes are coaxial cylinders, as in Fig. 1, $v = 2\pi(b + c)f\delta$.

Furthermore, P is always very small compared with unity unless the arc has already struck, as is evident from equation (1). Under these conditions

$$P_i = \frac{n\tau}{2\pi(b + c)f\delta Np}. \tag{2}$$

In order to apply this equation it is necessary to know $(b + c)$ and δ. Simple consideration of the distribution of potential (see Fig. 1) shows that

$$(b + c) = a\epsilon^{-\frac{V - V_r}{V}\log\frac{a}{c}}, \tag{3}$$

where a and c are the respective radii of anode and cathode.

The calculation of δ is more difficult, for it depends on the average distance which the electrons move beyond the point at which they have

acquired energy eV_r before making an effective impact, and also on the distribution of velocities of electrons which causes them to gain the critical energy at different distances from the cathode.

Suppose, first, that all n electrons are emitted from the cathode with

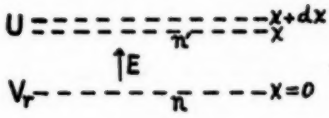

Fig. 2.

the same speed. Since all collisions at speeds less than the minimum radiating speed are perfectly elastic, they all attain this critical speed at the equipotential surface V_r whose position we shall specify by $x = 0$.

$$\nu = \frac{4rUp^2}{El^2} \tag{4}$$

is the number of collisions made by an electron, whose mean free path in the gas at unit pressure is l, in advancing unit distance.[1] E is the electric intensity and r is a numerical factor whose value lies between 0.87 and unity, and which may be neglected in our present work. If n' out of the n electrons reach the surface x, at potential U, without having lost their energy at an intervening impact, then $n'\nu dx$ of these will collide in the ensuing layer of thickness dx. The probability that any one of these impacts will be effective in activating a molecule is known to be, at least approximately, of the form $(U - V_r)/V_r = Ex/V_r$.[2] Thus the decrease in n' in passing through the layer dx is

$$dn' = -\frac{E}{V_r}\nu n' x dx,$$

whence

$$n' = n\epsilon^{-\frac{E\nu x^2}{2V_r}}.$$

The average distance which the electrons go beyond the surface V_r before colliding effectively is xdn'/n, integrated over all possible values of x, or

$$\bar{x} = \int_0^\infty \frac{E}{V_r}\nu x^2 \epsilon^{-\frac{E\nu x^2}{2V_r}} dx.$$

When this expression is integrated, and the value of ν substituted from equation (4), we find

$$\bar{x} = \sqrt{\frac{\pi}{8}}\frac{l}{p}, \tag{5}$$

which is about two thirds of the electronic mean free path.

[1] Benade and Compton, PHYS. REV., 11, p. 194, 1918.

[2] Compton, PHYS. REV., 15, p. 482, 1920.

This expression would give the appropriate value of δ (to at least the right order of magnitude) if all electrons left the cathode with equal speeds. Actually, however, they have initial speeds distributed approximately according to Maxwell's Law, and with an average energy $e\bar{V} = \alpha T$ characteristic of gas molecules at the temperature of the cathode.[1]

Thus the electrons, on the average, attain their critical energy eV_r within a layer whose thickness is $\delta' = \bar{V}/E$. Since this layer is distant $(b + c)$ from the axis,

$$E = \frac{V}{(b + c) \log a/c}.$$

Therefore,

$$\delta' = \frac{\alpha T}{eV}(b + c) \log \frac{a}{c}. \tag{6}$$

The value of δ, at least to a close approximation, is given by the sum of equations (5) and (6). An idea of the magnitude of $\bar{x}$ and δ' is obtained from the following calculations for mercury and helium, which have extremely small and large values, respectively, of the electronic mean free path.

TABLE I.

For Hg : $l = 0.0133$ cm.; $V_r = 4.9$ volts.
He : $l = 0.1175$ cm.; $V_r = 20.4$ volts.
Take $V = V_r + 1$, $a = 0.5$ cm., $c = 0.025$ cm.

p (mm.).	$\bar{x}$ (cm.).		$T°$ K.	δ' (cm.).	
	Hg.	He.		Hg.	He.
1	0.0083	0.0732	2000	0.0375	0.0150
2	0.0041	0.0366	2500	0.0470	0.0187
10	0.0008	0.0073	3000	0.0561	0.0225

It is seen that δ' is more important than $\bar{x}$ except at low pressures, particularly in the case of the heavier elements.

Returning to equation (2), we have for the final expression

$$P_i = \frac{n\tau}{2\pi(b + c)fNp\left\{\sqrt{\frac{\pi}{8}}\frac{l}{p} + \frac{\alpha T}{eV}(b + c) \log \frac{a}{c}\right\}}, \tag{7}$$

in which $(b + c)$ is given by equation (3). All quantities appearing here are known or can easily be measured except τ, which may tentatively be assigned a value of probably the right order of magnitude.

4. THEORY OF PHOTO-IMPACT IONIZATION.

On this theory, impacts in the shaded layer produce resonance radiation which is absorbed and reëmitted by atom after atom before escaping

[1] Sih Ling Ting, Roy. Soc. Proc. A., 98, p. 374, 1921.

from the gas, so that, as a result of each atom which is "activated" by a direct impact, many others are indirectly activated through absorption of radiation. Thus the probability P_r that an atom in the shaded region of effective impacts is in a partially ionized condition as a result of absorbed radiation is greater than the probability P_i of activation by direct impact. Our problem is to derive an expression for P_r.

It will be seen that the passage of radiation of the resonance type through the gas may be handled by expressions of the same type used to describe the diffusion of ions or of foreign molecules through a gas. A beam of light, passing through an absorbing or scattering medium falls off in intensity according to the law $I = I_0\epsilon^{-\alpha/x}$, where α is the absorption coefficient. A stream of particles passing through a gas is reduced by collisions, or scattering, according to the law $n = n_0\epsilon^{-x/l}$, where l is the mean free path. By analogy, let us call λ the mean free path of the radiation, defining it as the reciprocal of the absorption coefficient, or the distance in which the intensity of a beam falls to $1/\epsilon$ of its value as a result of resonance scattering. Then the average speed of the radiation $\bar{c}$ is this distance λ divided by the time of "activation" τ. We may then apply the diffusion equation

$$\int\int \frac{1}{3}\lambda\bar{c}\,\frac{\partial N'}{\partial n}\,dS = -\int\int\int R dx dy dz, \qquad (8)$$

where N' is the number of activated atoms per unit volume at any point in the gas, R is the net rate at which atoms are becoming activated by direct electron impacts and n is the outward normal to the closed surface over which the surface integral and within which the volume integral are taken.

This equation may have a mere formal, and not a physical, similarity to the case of diffusion of particles. However, there are grounds for quantelizing the radiant energy in units of magnitude $h\nu$, since it is known that this amount of energy is absorbed by an atom in becoming "activated" and it must be emitted again when the atom radiates and returns to the normal state. In this case the mean free path λ may be physically similar to the mean free path of a gas molecule. However this may be, it seems safe to use equation (8) to calculate N', which is our purpose.

Referring to Fig. 1, $n(1 - P)$ quanta of resonance radiation are produced by electron impacts each second in the cylindrical layer of radius $(b + c)$, length f and thickness δ. Some of this radiation diffuses toward the anode and some toward the cathode. Let the corresponding numbers of quanta be $n_a(1 - P)$ and $n_c(1 - P)$, respectively. There will be one

cylindrical surface within the layer δ across which the net radiation flux will be zero and which we may call the neutral surface. If we apply equation (8) to the region bounded by this surface and a co-axial surface of radius r, lying outside the neutral surface, we have

$$\frac{1}{3}\lambda\bar{c}\frac{dN'}{dr}(2\pi rf) = -n_a(1-P);$$

whence

$$N_a' = -\frac{3n_a(1-P)}{2\pi f\lambda\bar{c}}\log r + C_a$$

is the value of N' at any point outside the neutral surface, distant r from the axis. If the reflecting power of the electrodes for resonance radiation is negligibly small, as is approximately the case for most substances, $N_a' = 0$ when $r = a$, so that

$$N_a' = \frac{3n_a(1-P)}{2\pi f\lambda\bar{c}}\log\frac{a}{r}.$$

Similarly, at points inside the neutral surface,

$$N_c' = \frac{3n_c(1-P)}{2\pi f\lambda\bar{c}}\log\frac{r}{c}.$$

We have $n_a + n_c = n$ and at the neutral surface $r = (b + c)$ we have $N_a' = N_c' = N'$. From these relations it is easily shown that, at the neutral surface,

$$N' = \frac{3n(1-P)}{2\pi f\lambda\bar{c}}\frac{\log\dfrac{a}{b+c}\log\dfrac{b+c}{c}}{\log\dfrac{a}{c}}. \tag{9}$$

We implicitly assumed, in deriving this, that the thickness δ is negligible in comparison with the distance between the electrodes. This is nearly enough true in cases where low voltage arcs can be obtained. If the reflecting power of the electrodes for resonance radiation were considerable, equation (9) would take a more complicated form. It should, however, give results of the right order of magnitude in the cases to which we wish to apply it.

If the space charge around the cathode is negative, and if the voltage V is not much larger than V_r, then the critical layer δ is near the anode and we may replace a by $(b + c)$ in the denominator, so that

$$N' = \frac{3n(1-P)}{2\pi f\lambda\bar{c}}\log\frac{a}{b+c}.$$

Substituting for $(b + c)$ from equation (3) we have

$$N' = \frac{3n(1-P)}{2\pi f\lambda\bar{c}}\frac{V-V_r}{V}\log\frac{a}{c}. \tag{10}$$

If $1/\rho$ is the scattering coefficient of the gas at unit pressure for the resonance radiation, *i.e.*, ρ is the distance in which the intensity of a beam would fall to $1/\epsilon$ of its value, then p/ρ is the scattering coefficient at pressure p. If a fraction P of the atoms are partially ionized, and therefore unable to absorb resonance radiation, we have

$$\lambda = \frac{\rho}{p(1 - P)} \tag{11}$$

as the definition of the effective mean free path λ of the radiation. Putting $\bar{c} = \lambda/\tau$, we have

$$N' = \frac{3p^2 n\tau}{2\pi f\rho^2} \frac{V - V_r}{V} \log\frac{a}{c}, \tag{12}$$

since the factor $(1 - P)^3$ may be neglected owing to the small value of P possible by equation (1).

We can now derive P_r, the fraction of atoms in an activated state because of absorbed resonance radiation, for $P_r = N'/Np$, where Np is the number of molecules per unit volume. Thus, finally,

$$P_r = \frac{3pn\tau}{2\pi fN\rho^2} \frac{V - V_r}{V} \log\frac{a}{c}. \tag{13}$$

5. Importance of P_i and P_r.

The total probability of partial ionization in the region of effective impacts is $P = P_i + P_r$. It is found that P_r is much larger than P_i under conditions in which low voltage arcs are obtained, for

$$\frac{P_r}{P_i} = \frac{3p^2(b + c)\delta}{\rho^2} \frac{V - V_r}{V} \log\frac{a}{c}.$$

δ is the sum of $\bar{x}$ and δ' and is seen, in Table I., to be of the order of 0.01. $(b + c) \log a/c$ is of the order of unity in ordinary apparatus. If the arc strikes at about 0.1 volt above the minimum radiating potential, $(V - V_r)/V$ is of the order of 0.01. The pressures range from 0.5 to 10 mm. R. W. Wood [1] gives data from which ρ may be calculated to be 0.000675 cm. for the 2536 line of mercury vapor. Lamb [2] gives a very general law of resonance scattering, applicable if *every* molecule possesses an oscillator of proper frequency, from which it is readily shown that

$$\rho = \frac{1.54}{N\lambda_r^2}, \tag{14}$$

[1] Recent Researches in Physical Optics.

[2] Camb. Phil. Soc. Trans., 18, Stokes Commemoration, 1900.

where λ_r is the wave-length of the resonance radiation. This gives a minimum possible value of ρ which, in the case of the mercury 2536 line, is 10,200 times smaller than the experimental value, indicating that not all the molecules contain this type of oscillator or that their natural frequencies are distributed over a sufficient frequency range to account for the discrepancy. As far as the writer is aware, ρ has not been experimentally determined in any case except for the mercury 2536 radiation.

Using the experimental value of ρ for mercury, it is found that the ratio P_r/P_i is of the order of 10,000. Under any ordinary conditions, therefore, the direct effect of impacts is negligible in comparison with the indirect effect due to radiation in producing partially ionized atoms, capable of ionization by a low velocity impact.

The above comparison is possible without any knowledge of the time interval τ or the magnitude of the thermionic current. It is of interest, also, to see whether the probable *absolute* values of P_r or P_i are of the right order of magnitude to explain the size of the currents at which low voltage arcs are observed to strike.

Take the following reasonable values for the quantities involved. $a = 0.5$ cm., $c = 0.025$ cm. (20 mil wire), $f = 1$ cm. give the dimensions of the apparatus. $V_r = 4.9$ volts, $l = 0.0133$ cm. for mercury and $V_r = 20.4$ volts, $l = 0.1175$ cm. for helium are definite constants, as are also $N = 3.6(10)^{16}$ per c.c., $\alpha = 2(10)^{-16}$ ergs/degree and $e = 4.77(10)^{-10}$ e.s.u. $V = V_r + 1$ is a striking voltage for an arc in mercury or helium which is very easily realized with only moderate thermionic currents.

$\tau = 6(10)^{-7}$ sec. is the most favorable experimental value determined for any substance.[1] It was found for the D_3 line of helium. No determination of τ for a resonance line has ever been made. This is the most uncertain value entering into the calculations. $n = 1.84(10)^{14} V^{3/2}$, where V is in volts, gives the largest (hence most favorable) possible thermionic current. This is calculated from Langmuir's formula for the current limited by space charge in *vacuo*. Actually the current is less than this, since the presence of gas around the cathode reduces the rate of escape of electrons and thus increases the space charge for a given number of electrons. Furthermore, it is found experimentally that the currents need not be nearly so large as this to produce an arc at $V = V_r + 1$. Nevertheless, take this as the most favorable possible value.

$\rho = 0.000675$ cm. for mercury vapor, and is unknown, but probably smaller, for helium.

Substitution of these values in equation (7) for P_i gives the results in Table II.

[1] Stark, Ann. d. Phys., 49, p. 731, 1916.

TABLE II.

Values of P_i.

p (mm.).	$T°$ K.	Hg.	He.
1	2000	5.1 $(10)^{-7}$	12.6 $(10)^{-7}$
2		2.8	10.8
10		0.6	5.0
1	2500	4.7	12.1
2		2.6	10.0
10		0.55	4.2
1	3000	4.1	11.6
2		2.2	9.4
10		0.46	3.7

If these results are substituted in equation (1) we find n/n_0, or the ratio in which ionization has increased the total current just before the striking of the arc, as shown in Table III.

TABLE III.

Values of n/n_0.

p (mm.).	$T°$ K.	Hg.	He.
1	2000	1.00177	1.00061
2		1.00096	1.00052
10		1.00021	1.00024
1	2500	1.00161	1.00059
2		1.00089	1.00048
10		1.00019	1.00020
1	3000	1.00141	1.00056
2		1.00075	1.00045
10		1.00016	1.00018

These values are entirely inadequate to account for the experimentally observed values of n/n_0, which may range, roughly, from 2 to 100. Furthermore, they indicate variation both with pressure and temperature which is in a direction opposite to that observed.

R. W. Wood [1] has recently shown that, in the *excitation of fluorescence* of mercury vapor by light of wave-length 1850–2100 Å., there is a definite time interval of 1/15,000 second between the absorption of the light and the subsequent bursting into luminosity by the vapor. If this is the time interval which is effective in the present problem, we should put

[1] Roy. Soc. Proc. A., 99, p. 362, 1921.

$\tau = 6.7(10)^{-5}$ sec., which is about 110 times larger than the value assumed above. Substitution of this quantity gives values of P_i for mercury ranging between $5.6(10)^{-5}$ and $0.5(10)^{-5}$ and values of n/n_0 between 1.24 and 1.02 in the above tables. These values approach in magnitude the observed values of n/n_0, but are still too small. Furthermore, the assumed maximum value of n is about 200 times larger than actual values of n at which low voltage arcs can be obtained in mercury vapor. Taking this into account it seems as if the values of n/n_0 in Table III. represent the predictions of the Theory of Successive Impacts fairly accurately. In any case the indications are that successive impacts play a relatively insignificant part in cumulative ionization.

Turning now to photo-impact ionization and substituting the above values in equation (13) for P_τ, we find $P_\tau = 0.055p$ in the case of mercury vapor. No calculation can be made for helium because of lack of knowledge of ρ. At pressures at which a low voltage arc will strike in mercury, *i.e.*, above about 0.15 mm., this value of P_τ is more than sufficient to cause the arc to strike, as indicated by putting $n/n_0 = \infty$ in equation (1). For instance, at 1 mm. pressure, the calculated value is about 180 times as large as necessary to produce the arc. In other words, the theory would predict an arc at a thermionic current at least 180 times less than the maximum possible current which was assumed above. This is in good agreement with observations in an experiment, still in progress, by Mr. Y. T. Yao and the writer. 1/180 part of the value of n assumed above is $1.46(10)^{13}$, which corresponds to a thermionic current of 2.2 micro-amperes. In an apparatus of the dimensions assumed above, and under approximately the assumed conditions, the arcs strike at currents of a few micro-amperes.

Thus it appears probable that the ionization observed in gases at voltages less than their minimum ionizing voltages is due chiefly to photo-impact ionization. We cannot say more than this until the constant τ is measured for the resonance radiation of those gases in which low voltage arcs can be obtained, *i.e.*, monatomic gases and vapors.[1]

[1] *Note.*—Hughes has pointed out the fact that, in a gas whose minimum radiating potential is less than half the ionizing potential, no two cumulative effects at this voltage could contribute sufficient energy for ionization. The experiments of Mr. Yao and the writer indicate, however, that proper corrections for velocity distribution place the striking voltage of the low voltage arc in mercury at about 5.6 volts, instead of 4.9 volts. Thus the electron impact against the partially ionized atom must contribute an amount of energy sufficient to increase the total potential energy to the value required for ionization, as demanded by theory. Furthermore, experiments to be later described by Mr. Yao indicate that the low voltage arc in mercury may be dependent on either the 4.9 volt (2356) or the 6.7 volt (1849) radiation according to whether the vapor is in close or distant communication with its liquid surface. This and spectroscopic evidence suggests two meta-stable states of the neutral mercury atom. Thus the actual conditions are somewhat more complicated than those assumed in the theory.

6. IONIZATION BY SUCCESSIVE PHOTOELECTRIC ACTION.

F. Horton and Miss A. C. Davies have recently suggested that the second stage in the process of ionization may also be accomplished by the absorption of radiation from neighboring impacts.[1] Their experimental results with helium prove the diffusion of resonance radiation through the gas, the existence of photo-impact ionization and, apparently, the existence of ionization by photoelectric action on atoms which are already partially ionized by absorption of radiation. If there exists a second type of resonance radiation which is capable of ionizing atoms which have partially absorbed the first type, or if the resonance potential exceeds half the ionizing potential, we should expect this radiation to be more effective than electron impacts in completing ionization, for the same reason that radiation is more effective than impacts in producing the first stage of ionization. An attempt to reduce this type of ionization to quantitative form led to some suggestive results, but the equations become so complicated and the relation of the second photoelectric process to the first involves such uncertain hypotheses that it will not be discussed here. The importance of this type of ionization, which seems proved in the case of helium, can probably best be established in the case of metallic vapors by direct experiment or by a proven inadequacy of the two alternative processes discussed in this paper.

7. DISCUSSION.

The chief result which seems to emerge definitely from the above discussion is that resonance radiation plays a very important part in ionization at low voltages. At high voltages and low gas pressures, as in the Geissler tube discharge, there is good opportunity for ionization by direct single impacts. But if the potential drop in the distance of a mean free path is much less than the ionizing potential, conditions are unfavorable for direct ionization and the cumulative type of ionization preponderates.

If we carry conditions a step further in the same direction we have the case of purely temperature ionization, in which there is no potential drop at all. Here, presumably, cumulative ionization is practically the only type and resonance radiation is an essential agent in the process.

This concept has an interesting application to the case of ionization in the solar chromosphere, recently discussed by Saha.[2] He treats the chromosphere as an enclosure at constant temperature and uses Nernst's equation of the isobar to calculate the degree of ionization of any gas,

[1] Phil. Mag., 12, p. 746, 1921.

[2] Phil. Mag., 40, pp. 472, 809, 1920.

treating ionization as a dissociation. This theory has been remarkably successful in explaining and predicting features of the solar spectrum. The theory fails, however, to explain the fact that, although sodium and barium have equal ionizing potentials, barium is much more completely ionized than is sodium. This may be explained, as has been pointed out by Professor H. N. Russell and the writer,[1] by taking account of the effect of resonance radiation of these elements. For the chromosphere is not an enclosure, but a layer through which is passing an enormous flux of energy from the hotter interior. Observation shows, and there are reasonable explanations of it, that this flux is particularly deficient in radiation of the resonance type for sodium,—much more deficient than in the corresponding radiation for barium. Thus neutral barium atoms must exist largely in the partially ionized state. This is equivalent to a relative lowering of the ionizing potential for barium, permitting more complete ionization.

The spectral peculiarities of light excited in the low voltage arc (and also in the high temperature furnace) are necessary consequences of the mechanism of ionization here discussed. If the gas pressure is high and the electric field weak, so that the energy gained by an electron between collisions is small, there is little chance of exciting any radiation except that of the resonance type corresponding to the inelastic impact of lowest velocity. Thus we get the "single line" spectrum. If the pressure is less or the field greater, an appreciable number of electrons may attain speeds sufficient to excite a second type of resonance radiation of shorter wave-length. Such lines are the $1S - 1p$ and $1S - 1P$ lines of the alkaline earths. As a result of cumulative ionization, a complete spectrum will be present, but will be weak and will become, relatively to the resonance line or lines, of vanishingly small intensity as the voltage and current are diminished.

As soon as the arc strikes, conditions are at the other extreme. Owing to the positive space charge around the cathode, the electric intensity in this region is very large and the electrons are shot out into the gas with practically the maximum velocity attainable at the applied voltage. Then the resonance lines are not particularly "favored," and the entire arc spectrum appears. If the voltage is sufficiently large, the enhanced lines will appear. In fact, in this type of arc, there is realized somewhat the same distribution of potential which has been secured in a more manageable way by Foote, Meggers and Mohler[2] by the introduction of an additional gauze electrode near the cathode, and which has made possible their most interesting work on enhanced spectra.

[1] Astrophys. Jour. (in print).

[2] Phil. Mag., 42, p. 1002, 1921.

In conclusion, mention should be made of a method of experimentally testing the above theories. The most decisive test of photo-impact ionization would be to illuminate a gas with its resonance radiation from an external source and see whether ionization can then be observed with electron impacts of velocity less than the ordinary minimum ionizing velocity. Such an effect has been observed in iodine vapor,[1] but this is a case of fluorescence and not true resonance—though the mechanism of ionization is probably similar. Last year, Mr. Yao made careful attempts to prove photo-impact ionization in this way with mercury and sodium vapors, but with results which were indecisive because of peculiar difficulties in getting intense resonance radiation of exactly the effective wave-length into a gas from an external source. These experiments are now being repeated under more favorable conditions.

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Since this article was submitted, a very interesting paper by F. M. Kannenstine (Astrophys. Jour. 55, p. 345, 1922) has described a method of determining the time during which the metastable (partially ionized) helium atoms persist in a helium arc after the exciting voltage has been removed. The time is about 2.4 $(10)^{-3}$ second, which is much larger than the value of τ used above. If cumulative ionization is by successive impacts, then Kannenstine's value should be the value of τ for helium. If the ionization is of the photo-impact type, however, Kannenstine's measurements give the time required for the resonance radiation to escape from the region of effective impacts, which may include the time of many absorptions and reëmissions, and therefore τ would be much smaller than 2.4 $(10)^{-3}$. This point, and other recent evidence will be discussed briefly in another communication.

[1] Smyth and Compton, PHYS. REV., 16, p. 501, 1920.

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THE THERMIONIC WORK FUNCTION OF TUNGSTEN.

BY C. DAVISSON AND L. H. GERMER.

SYNOPSIS.

Measurements of the Thermionic Work Function of Pure Tungsten have been made by two methods for the same segment of a uniformly heated filament simultaneously. The filament, of diameter 0.0755 mm. and with three finer tungsten leads welded to it 3.85 cm. apart, was kept stretched, by means of a molybdenum spring, between two molybdenum plates used to apply an electric field and to receive the space current, the ends of the filament being surrounded by guard boxes to limit the emission to the section of the filament between the potential leads. All parts were sealed inside a tube and so thoroughly out-gassed by alternately baking the tube and heating the metal parts that the residual pressure during the measurements was only about 10^{-7} mm., as measured with an ionization manometer. (1) In the *calorimetric method*, the equivalent voltage of the work function was computed from the relation $\varphi = 2EI(\Delta E/i)/(E - IdE/dI)$, where E is the voltage across the segment and I the current through it, and ΔE is the voltage change when the space current i flows from the filament, I being kept constant. Since ΔE was only from 10^{-3} to 10^{-4} times E, special precautions had to be taken to measure it with sufficient accuracy and to keep I sufficiently constant. Observations from 2070° K. to 2300° K. show an apparent increase for φ of about 2 per cent. in this range, considerably greater than is to be expected theoretically. At 2270° K., when correction is made for the asymmetry of the electric shielding ($-$ 2.24 per cent.) and for the radiation from the plates heated by the space current ($+$ 4.12 per cent.), φ comes out 4.91 $\pm$ .05 volts. (2) In the *temperature variation method*, assuming Richardson's equation $\log i = \text{const.} + 1/2 \log T - \varphi e/kT$, measurements of the emission in the range 1930° to 2300° K. give for φ the values 4.87 or 4.78 volts according as the temperature scale of Langmuir or that of Worthing and Forsythe is adopted. The latter scale is probably more reliable but gives a value of φ 2.7 per cent. less than the value obtained calorimetrically, a difference believed to be greater than the probable error of the measurements.

Suggested Modification of the Theory of Conduction of Electricity in Metals.—The thermal energy of conduction electrons is supposed to be $3/2kT$ by the classical theory upon which the above computations of φ are based. If, however, the energy is taken to be practically zero then the same data lead to 4.52 and 4.48 volts for the values of φ by the two methods and the agreement is within the probable error of the measurements.

INTRODUCTION.

THE equation,

$$i = AT^{1/2}\epsilon^{-b/T}$$

proposed by Richardson[1] in 1902 to represent the current from a thermionic source as a function of temperature has been found to be of general and precise application within the limits of error of such measurements.

[1] Camb. Phil. Soc. Proc., 11, p. 286 (1902). Phil. Mag. A, 201, p. 497 (1903).

This equation is a special form of the more general equation

$$i = AT^{1/2}\epsilon^{\int^T \frac{\omega}{kT} dT},$$

in which ω is the work required to liberate a single electron and k is Boltzmann's gas constant. The first equation is obtained from the second by assuming that ω is independent of T. The integral in the general equation can then be evaluated, and the relation takes the simpler form, in which $b = \omega/k$. It is customary to express ω as the product of a voltage times the electronic charge, $\omega = \varphi e$, so that $\varphi = kb/e$. This is referred to as the equivalent voltage of the work function.

The adequacy of the simpler equation might seem to justify the assumption regarding the independence of ω and T, and this, within limits, is true. It is well recognized, however, that on account of the particular form of the temperature-emission relation moderate variation in ω with temperature cannot be detected within the limits of accuracy ordinarily attained in such measurements. Thus, if $\omega = \omega_0(1 + \alpha T)$, the general equation reduces to

$$i = AT^{\left(\frac{1}{2}+\frac{\omega_0\alpha}{k}\right)}\epsilon^{-\frac{\omega_0}{kT}}$$

and the only hope of detecting the variation in ω is to distinguish between $1/2$ and the true value of the exponent of T. This T factor is so unimportant compared with the exponential factor that its exponent cannot be determined with anything approaching the required accuracy. If the exponent $1/2$ is used, and ω is not independent of T, then the experimentally determined value of b corresponds to an average value of ω over the temperature range of the measurements. It is possible, however, that the average value of ω so determined may differ appreciably from the average value of the true work function.

Apart from all differences arising from experimental error, real differences might be expected owing to the untenable nature of one or more of the assumptions involved in the derivation of Richardson's equation. The argument by which the equation is developed is essentially thermodynamic, yet the equations of thermodynamics have to do only with states of equilibrium, and in particular do not deal with such quantities as rates of emission. To arrive at the relation between rate of emission and temperature it is necessary to assume (1) that the laws of the kinetic theory of gases may be applied to an atmosphere of electrons in equilibrium with a thermionic source, (2) that the coefficient of reflection of electrons from the surface of such a source is zero, or at any rate, independent of temperature, and (3) that the rate of emission of electrons at

a given temperature is as great when all emitted electrons are being drawn off as fast as liberated, as when the system is in equilibrium, or, at least, that there is a fixed ratio between these rates. The failure of any one of these conditions may lead to incorrect estimates of the thermionic work function calculated from the b constant of Richardson's equation, without necessarily affecting the appropriateness of the equation to represent the temperature-emission relation. Thus there is some theoretical importance attached to direct comparisons between calorimetrically determined work functions and corresponding values of the quantity, kb/e, obtained from temperature-emission data.

Calorimetric determinations of the work function may be made in the case of thermionic emission by measurements on what is generally referred to as the cooling effect. Such measurements were first attempted by Wehnelt and Jentzsch [1] working with lime coated filaments. The experimental conditions under which these investigators worked appear to have been too far from ideal, however, to permit of their obtaining any satisfactory results. The first successful measurements were made by Cooke and Richardson on osmium filaments[2] and later on tungsten filaments.[3] Other measurements have since been made for tungsten, carbon, molybdenum and tantalum by H. H. Lester.[4] The values of φ determined in these experiments have ranged from 4 to 6 volts, and are, therefore, in approximate agreement with the values of φ determined for these same materials from their values of b. In the case of tungsten Lester found that his value of φ determined by the cooling method agreed to within 0.7 of one per cent. with a value previously calculated by Langmuir from the value of b determined for a filament from the same batch of pure tungsten. While this agreement is well within the limits of experimental error there is some question as to how much importance should be attached to it. The thermionic properties of metals are appreciably altered by very slight gaseous or other contamination; and for this reason it is doubtful if a comparison based on different filaments, even though they be from the same batch of metal, can ever be regarded as satisfactory.

Direct comparisons of values of φ determined in the two ways were made in 1917 by Dr. Wilson[5] of this laboratory for the emission from various oxide coated filaments. While these experiments were not designed as a critical study of the thermionic work function as such,

[1] Ann. d. Phys., 28, p. 537 (1909).
[2] Phil. Mag., 25, p. 624 (1913).
[3] Phil. Mag., 26, p. 472 (1913).
[4] Phil. Mag., 35, p. 197 (1916).
[5] Nat. Acad. Sci., Proc. 3, p. 426 (1917).

the values found for individual filaments by the two methods were in very good agreement.

The purpose of this paper is to present results obtained by us in the measurement of the work function by the two methods simultaneously upon a single filament of pure tungsten. The calorimetric measurements are described in Part I., the temperature-emission measurements in Part II., and this is followed by a discussion of the results of the two sets of measurements.

Part I. Calorimetric Measurement of φ.

A. Theory of the Measurement.

The general scheme of the experimental arrangement used in these measurements is shown in the circuit diagram, Fig. 1. The filament to be tested, *FF*, is provided with potential leads at *pp*, far enough removed from the ends of the filament to avoid effects arising from end cooling, and means are provided (not indicated in the diagram) for limiting the emission of electrons from the filament to the region between these points.

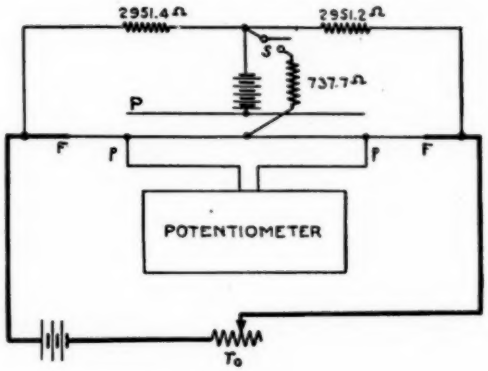

Fig. 1.

With the plate *P* at a negative potential and a heating current of *I* amperes through the filament the difference of potential between the points *p* is, say, *E* volts. We have then for the resistance of the filament, $R = E/I$ ohms, and for the net radiation, $W = EI$ watts. It is assumed that *R* and *W* are functions of temperature only, and in particular that they are independent of whether or not electrons are being drawn from the filament. This does not apply in general to the total power supplied to the filament but only to that portion of the total power which goes to maintain thermal radiation. *W* may then be regarded as a function of

R, and any small change in R is accompanied by a small change in W such that

$$\Delta W = \frac{dW}{dR}\Delta R = I^2 \frac{I\frac{dE}{dI} + E}{I\frac{dE}{dI} - E}\Delta R.$$

If now the plate P is switched to a positive potential an electron current of i amperes, which for the present will be supposed small compared to I, flows from the filament and simultaneously the heating current becomes $I + \Delta I$ and the voltage drop $E + \Delta E$.

We must now distinguish carefully between $\overline{W}$ the total power supplied to the filament and W the power radiated from it. Without space current these are equal. When the space current is switched on, however, they are altered in different amounts. The increase in $\overline{W}$ is given by

$$\Delta\overline{W} = I\Delta E + E\Delta I,$$

while the increase in W must be calculated from the change in resistance of the filament and the known rate of variation of W with R. Thus $\Delta R = (I\Delta E - E\Delta I)/I^2$ and

$$\Delta W = \left[\frac{I\frac{dE}{dI} + E}{I\frac{dE}{dI} - E}\right](I\Delta E - E\Delta I).$$

The difference between these two increments is the power absorbed by the thermionic emission. Expressing this as a voltage, φ, times the emission, i, we have,

$$\varphi i = \Delta\overline{W} - \Delta W = \left[\frac{2EI}{E - I\frac{dE}{dI}}\right]\left(\Delta E - \frac{dE}{dI}\Delta I\right).$$

In using this relation to determine φ it has been found convenient to compensate the change in filament resistance by an appropriate alteration in the resistance r_0, so that ΔI is made zero. φ is then calculated from the expression,

$$\varphi = \left[\frac{2EI}{E - I\frac{dE}{dI}}\right]\frac{\Delta E}{i} = Q\frac{\Delta E}{i}.$$

In order that this formula shall yield the true value of φ the value of ΔE employed must be the voltage change actually due to the cooling

effect of the emission. In addition to this true ΔE, the existence of emission gives rise to several other voltage changes across the filament. A considerable share of the first part of this paper is taken up by a consideration of these spurious changes. Some of these reverse sign relative to the true effect with reversal of the heating current and are eliminated from the results by averaging observations made with the two directions of heating current. Others, however, do not reverse sign in this way.

One such effect that requires special attention results from the change in temperature of the plates under electron bombardment. A part of the additional radiation from the plates is absorbed by the filament giving rise to an increase in filament temperature and resistance, and in consequence to a change in voltage across the filament. Another cause of error arises from the dissymmetry of the potential field. The emission of electrons from the filament is limited to the portion between the potential leads by electrostatic guard boxes surrounding the ends of the filament and maintained a few volts negative to it. On account of the drop in potential along the filament the relations between plate, guard and filament potentials are not the same at the two ends of the filament. This gives rise to a slight shift of the center of emission toward the negative end and to a voltage change that is unrelated to the cooling. Both of these effects are important even when i is small in comparison with I.

If i is not small compared with I still another spurious voltage change becomes important. This results from the addition of the returned space current to the normal heating current with the consequent variation of total effective heating current from point to point along the filament. We shall begin by considering this last effect somewhat in detail.

The total potential drop across the filament is always

$$E = \int_{-\frac{1}{2}L}^{\frac{1}{2}L} \rho I dx,$$

where L is the total length of the filament (between potential leads) and ρ is the resistance per unit length. The variation of this integral is

$$\Delta E = \int_{-\frac{1}{2}L}^{\frac{1}{2}L} (\rho \delta I + I \delta \rho) dx.$$

If we disregard for the time the effect of cooling then $\rho = f(I)$, and

$$\delta \rho = \frac{d\rho}{dI} \delta I = \frac{d\rho}{dI} i,$$

where i now represents the space current flowing in the filament at the point x. Thus the change in voltage under consideration is given by

$$\Delta E' = \rho' \int_{-\frac{1}{2}L}^{\frac{1}{2}L} i dx,$$

where $\rho' = \rho + I(d\rho/dI)$.

To obtain an expression for i as a function of x we have the following relations:

(1) $$\omega = \rho I^2 + I\left(2\rho + I\frac{d\rho}{dI}\right)i \text{ watts}$$

where ω is the power radiated per cm. of filament.

(2) $$\omega = \pi d\epsilon\sigma T^4 = \pi d\epsilon_0\sigma T^5 \text{ watts},$$

where d is the diameter of the filament in cm., σ is the Stefan-Boltzmann radiation constant, and $\epsilon = \epsilon_0 T$ is the total thermal emissive power of tungsten, which is closely proportional to T over the range of our measurements.

(3) $$S = -\frac{di}{dx} = \pi d A T^{1/2}\epsilon^{-b/T},$$

where S is the emission per cm. of filament, and A and b are the constants of Richardson's equation. Here and in what follows the positive direction of current is taken to coincide with the direction of flow of electrons, and the sign of potential gradient along the filament is also reversed.

Combining the above relations it can be shown that, to a close approximation, the value of i at the point x is

$$i = \frac{1}{\beta}\log\frac{1}{\beta S_0 x + C},$$

where S_0 is the emission per cm. of filament for heating current I

$$\beta = \frac{b\left(2\rho + I\frac{d\rho}{dI}\right)}{5\rho I}\left\{\frac{\pi d\epsilon_0\sigma}{\rho I^2}\right\}^{1/5}$$

and C is a constant of integration.

If the current flowing into the filament at the point $-\frac{1}{2}L$ is $(I + i_1)$ and that flowing out of the filament at the point $\frac{1}{2}L$ is $(I + i_2)$, then

$$i = i_0 - \frac{1}{\beta}\log\left(\cosh\beta i' + \frac{2x}{L}\sinh\beta i'\right),$$

where

$$i_0 = \frac{1}{2}(i_1 + i_2) \qquad \text{and} \qquad i' = \frac{1}{2}(i_1 - i_2).$$

Substituting this value of i into the expression for $\Delta E'$, and integrating, we find

$$\Delta E' = \rho' L\left[i_0 + \frac{1}{\beta}(1 - \beta i'\cosh\beta i')\right]$$

or to a close approximation

$$\Delta E' = \rho' L\left(i_0 - \frac{\beta}{3}i'^2\right).$$

If we now, as formerly, use i to represent the total emission from the filament, we have, for the case in which the space current is returned equally to the two ends of the filament, $i_1 = -i_2 = 1/2i$ so that $i_0 = 0$ and $i' = 1/2i$, and

$$\Delta E' = -\frac{\beta}{12}\rho' L i^2.$$

It is to be noted that the voltage change is, in this case, negative. It is of the same sign as the change due to the cooling effect, and results, if not allowed for, in too great a value of the work function.

If the space current instead of being returned to the ends of the filament, as in the case just considered, is returned to the middle point of the filament, conditions are considerably altered. In computing the spurious ΔE for this case the total effect is obtained by adding the effects computed for the two halves of the filament separately. In making this computation it must be borne in mind that when the space current is returned to the middle point of the filament it does not divide equally between the two halves, but that the greater part of it flows from the middle point in the direction of flow of the main heating current. When this dissymmetry is taken into account one finds for the total spurious voltage change in this case

$$\Delta E' = \frac{\beta}{24}\rho' L i^2.$$

The change is positive instead of negative and in magnitude it is one half as great as the $\Delta E'$ for the current returned equally to the ends of the filament. This suggests the possibility of so arranging the return of space current to the filament that the voltage change due to redistribution of the heating current is completely eliminated. It turns out from a deduction similar to those already indicated that to make $\Delta E'$ zero the current returned to the middle point must be four times as great as the current returned to each end. This condition is obtained in the circuit of Fig. 1 when the switch, S, is closed.

Measurements may also be made for the space current returned equally to the ends of the filament, and entirely to the middle of the filament. If the apparent values of the work function obtained in these ways are, respectively, φ_1 and φ_2, then the true work function is given by $\varphi = \frac{1}{3}(\varphi_1 + 2\varphi_2)$. This again avoids having to calculate the spurious ΔE.

Another source of error mentioned above is that due to the change in temperature of the filament that results from the heating of the plates under electron bombardment. When the space current is switched on the power, $w = Vi$, where V is the plate potential, is dissipated at the

plates, and when the system has come to equilibrium the thermal radiation from the plates is increased by this amount. A fraction, f, of this radiation is absorbed by the filament, the value of f being determined by the geometry of the system and by the coefficient of absorption of the filament for the radiation reaching it. In calculating the error that this "back heating" introduces into the results we may disregard for the time the voltage changes due to all other causes.

When the space current is switched on the power radiated from the filament changes from $W = EI$ to $W + \Delta W = I(E + \Delta E'') + fw$, and the resistance of the filament changes from $R = E/I$ to $R + \Delta R = (E + \Delta E'')/I$, since I is kept constant.

From these relations we have

$$\frac{\Delta W}{\Delta R} = \frac{dW}{dR} = I^2 \left[\frac{I\frac{dE}{dI} + E}{I\frac{dE}{dI} - E} \right] = \frac{I\Delta E'' + fw}{\Delta E''/I}.$$

Solving this for $\Delta E''$

$$\Delta E'' = \frac{-fw}{\left[\frac{2EI}{E - I\frac{dE}{dI}} \right]} = \frac{-fVi}{Q}$$

or, since

$$\varphi = Q\frac{\Delta E}{i},$$

$$Q\frac{\Delta E''}{i} = \Delta\varphi = -fV,$$

i.e., the back heating causes an error in φ which is equal to $-f$ times the plate voltage. This relation is particularly interesting in showing that this error is independent of the magnitude of the space current.

The most satisfactory means we have found of correcting for this error is to observe the value of $\Delta E/i$ at constant filament current for a series of plate voltages, and then to extrapolate to the value of $\Delta E/i$ corresponding to plate voltage zero.

It seems advisable to omit consideration of effects arising from dissymmetry of the potential field, mentioned above, from this section as they can be dealt with more easily in connection with the experimental results.

As to the spurious voltage changes that reverse sign with reversal of the heating current, these result from geometrical dissymmetry and from variation in thermionic properties from point to point along the

filament. It can be shown quite generally that all first order errors arising from these causes are eliminated from the results by averaging observations for the two directions of heating current.

B. Description of Tube and Circuit.

All satisfactory measurements were made upon a filament of very pure tungsten kindly supplied to us by Dr. Langmuir of the General Electric Company. The measured diameter of this filament was $(7.595 \pm .015) \times 10^{-3}$ cm. To it were welded at equal intervals three very fine tungsten leads of measured diameter 1.55×10^{-3} cm. These were to serve as potential leads and as space current return to the middle of the filament. The welding was performed under a microscope in an atmosphere of nitrogen. The fine leads were spaced as carefully as possible, and after welding the distances between leads were measured to be $3.849 \pm .001$ cm. and $3.842 \pm .001$ cm., respectively. At each point the fine filament was welded straight across the larger filament, so that there were really two leads attached at each of the three points.

The filament with fine leads attached was mounted as shown in Fig. 2. The lower end F_1, about 2.5 cm. from the potential lead P_1, was welded to a heavy platinum lead which passed directly through the press of the tube. The upper end of the filament F_2, about 2.5 cm. from the top potential lead P_2, was welded to a heavy platinum wire which in turn was supported from a glass arbor of the tube by a spiralled molybdenum spring. This heavy platinum wire passed through two small holes in a nickel guide, and in this way the tungsten filament was maintained straight and taut at all temperatures. The current lead to the top of the filament consisted of three flexible platinum ribbons welded to the heavy platinum wire below the nickel guide. These ribbons passed to a platinum lead-wire sealed through the top of the tube.

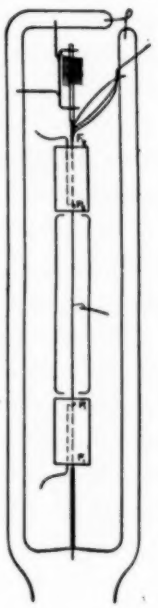

Fig. 2.

Heavy tungsten wires were welded to the fine tungsten potential leads about 1 cm. from the filament, and these in turn were welded to supporting wires and lead-in wires sealed through the bulb. The purpose of interposing tungsten wires adjacent to the potential leads was to avoid, as far as possible, the development of thermo-electric forces in the potentiometer circuit.

The space current leads to the middle of the filament, which were not to be used for potential leads, were fastened directly to nickel supports and finally led out through a platinum wire in the press.

The plates for receiving the space current and a pair of guard boxes for limiting the emission to the section of filament between the potential leads were made of molybdenum, and arranged as indicated in the figure. The plates were each 2.5 cm. by 7 cm., and bent inward slightly at top and bottom as shown. When mounted they were 1.25 cm. apart and symmetrically located with respect to the filament. The boxes were 2.5 cm. × 2.5 cm. × 1.25 cm., open toward the filament supports at the respective ends and closed at their opposite sides except for centrally located circular holes about 1 mm. in diameter through which the filament passed.

In order to free the parts from gas as much as possible, the glass parts were baked for several hours in a vacuum furnace at about 500° C. before assembling, and all of the metal parts, with the exception of the molybdenum spring and the tungsten filaments, were pre-glowed in vacuo at a high temperature. On the pumps the main tube and a Buckley ionization manometer which was sealed to it were baked out at 400° C. The molybdenum plates were raised to white heat by electron bombardment from auxiliary tungsten filaments mounted outside the plates, and the guard boxes were heated to about 1500° C. by Foucault currents induced from a high frequency current in a solenoid surrounding the tube. The procedure of alternately baking the tube and heating the metal parts, as described, was repeated several times. The tube was finally sealed from the pumps at a pressure of about 1×10^{-6} mm. Hg with the molybdenum plates at a white heat.

None of the final measurements were made until the tube had been in circuit over a month. During all this time some of the filaments were heated and for a large part of it all the filaments were hot and furnishing space currents. The final pressure obtained with the main filament cold was below 5×10^{-8} mm. Hg and was scarcely measurable. With the main filament at its highest temperature the pressure was about 5×10^{-7} mm. Hg.

Throughout these final measurements the main filament and one of the two manometer filaments, which were of tungsten, were glowed continuously. The main filament was cold for only a few minutes about twice a week while the storage batteries were being changed. This precaution was found necessary to insure extreme steadiness of emission. By taking this precaution measurements made at any working temperature under identical conditions seldom differed by as much as 2 per cent.

after an interval of several weeks. Under the ordinary working conditions unsteadiness of emission was never a disturbing factor.

All the essential measurements were made with a Leeds and Northrup potentiometer. As the maximum potential between the potential leads was about 12 volts the potentiometer could not be used in the normal way. In place of the customary 2-volt storage cell a battery of 18 volts was used across the potentiometer and the current adjusted to balance a Weston Standard Cell across a single 5-ohm coil. With this arrangement the potentiometer readings could be reduced to volts by multiplying by the factor 10.1870, and the maximum range of the instrument was about 16.3 volts instead of the usual 1.60 volts.

The accuracy of the potentiometer was checked by comparing together four coils taken at random and by comparing a coil with the slide wire. The coils were found to be identical but the slide-wire had a slightly low resistance. A reading on the slide-wire could be reduced to a "coil reading" by dividing by 1.0014(5). This ratio was checked for several different points on the slide-wire, and it was found to hold when the potentiometer was balanced in the normal way as well as when it was balanced with the standard cell across one coil. Correction for this ratio was made before using any reading taken on the potentiometer.

The voltage between the potential leads was measured directly on the potentiometer. The potentiometer was also used in determining the heating current and the space current from the filament, in the first case from measurements of the voltage across a standard ohm in series with the filament, and in the second from the voltage across a variable resistance dial box through which the space current flowed. The steps of this dial box were measured and found to be sufficiently accurate. The certificate of Leeds and Northrup gave the resistance of the standard ohm as .99997 International ohms at 25° C. No correction was made for its resistance not being exactly one ohm.

The arrangement just indicated suffices for taking the points of the $E - I$ curve and from them determining the function,

$$Q = \frac{2EI}{E - I\frac{dE}{dI}},$$

for the different values of I at which the cooling is to be measured. It is also satisfactory for measuring the emission as a function of I. The temperatures can be found from the $E - I$ curve together with the known temperature characteristics of tungsten and thus all the data are available for finding the constants of Richardson's equation.

The measurement of ΔE, which is the essential datum in determining φ, cannot be satisfactorily carried out with the arrangement described. The actual magnitude of ΔE for our filament was always in the neighborhood of 2 volts per ampere of space current. Thus, in order to measure ΔE correct to 1 per cent. for an emission of 1×10^{-3} amperes, it would be necessary to determine the voltage change represented by ΔE correct to 2×10^{-5} volts, or approximately 1/50 of a division on the slide-wire; the total ΔE would correspond to only 2 divisions. Since dE/dI for the filament was about 20 volts per ampere, an error of 1/10 of one division in the resetting of I would involve an error of 100 per cent. in ΔE. Also slow time changes in the filament heating current and the potentiometer current always occurred, and it was thus necessary to measure E very quickly indeed after setting these currents to their correct values.

The necessary accuracy in resetting the filament current was finally obtained by placing in series with the filament a fixed resistance of such magnitude that the desired heating current was established by making the voltage across this resistance equal to that of the standard cell. (An appropriate series of heating currents was obtained by using an appropriate series of such fixed resistances.) By this procedure the accuracy of resetting the heating current is not made to depend upon the accuracy of resetting the potentiometer. Great care was taken for maintaining the heating current and the potentiometer current steady, and rapid manipulation was aided by a convenient arrangement of switches and regulating resistances.

After these factors had been satisfactorily taken care of, it became feasible to increase the sensitivity of the potentiometer for directly reading the change in E. This was accomplished by applying the 18-volt potentiometer battery across the potentiometer in series with two variable external resistances (a and b in Fig. 3). The standard cell was balanced against the potentiometer in the normal way, the bulk of the 18 volts being taken up across these resistances. The circuit was arranged to balance the voltage between the filament potential leads across one of the external resistances and part of the slide-wire. With this arrangement the values of these resistances were not required as only changes in E were measured and these changes fell well within the limits of the slide wire. The potentiometer sensitivity is now normal.

With these improvements in the circuit ΔE could be determined with a satisfactory accuracy for as low a space current as 0.6×10^{-3} amperes. The practical upper limit to the emission was about 9.5×10^{-3} amperes. It was planned to determine ΔE, and thus φ, for eight different filament temperatures including the temperatures which gave these two extreme

values to the emission. For this purpose it was necessary to have the fixed resistance in the filament circuit adjustable to eight values which would determine these required temperatures. This resistance was made of a length of No. 17 "Ideal" resistance wire wound upon a copper tube. It was divided into four sections, three of which could be shunted by heavy amalgamated copper bars dipping into cups of mercury. By different combinations of these shunts eight different fixed resistances were available. These resistance steps were very constant, no variation in any of them as great as .005 per cent. being found during a period of fourteen months.

The essential features of the circuit have already been described. For the sake of completeness a simplified diagram of the circuit showing the more important switches is given in Fig. 3. The arrangement for re-

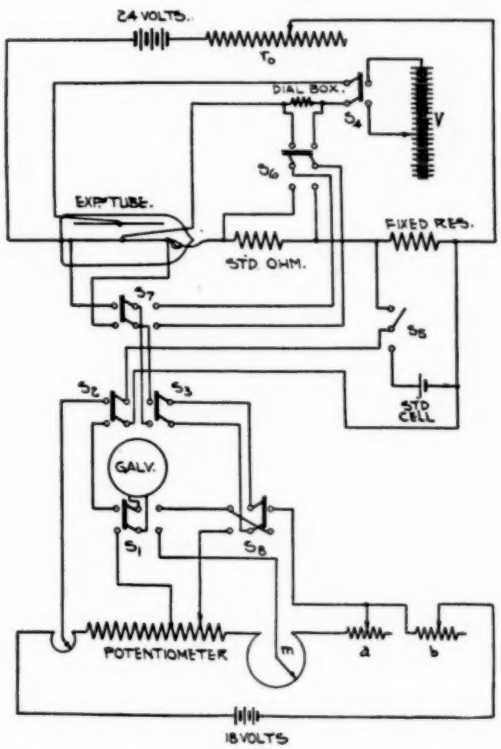

Fig. 3.

turning the space current to the filament in the three different ways is not shown, nor are the means for using the 18 volts straight across the potentiometer and balancing the standard cell across one coil. The electrostatic shields which limit the emission from the filament are not

shown. Means were provided for varying the potentials of these two shields independently. The space voltage, V, could be varied in steps of one dry cell from one cell to a voltage of 500, and it could be applied in either direction between the filament and plate. A reversing switch was inserted in the filament circuit so that observations could be made for both directions of heating current. This necessitated reversing switches for the potentiometer current, for the galvanometer, for the standard cell, and for the potential leads from the space current dial box. These five reversing switches were arranged to be operated simultaneously. There were other switches for increasing the convenience and accuracy of measurements, but these need not be considered here.

It may be instructive to trace the manipulation of switches in a typical measurement of $\Delta E/i$. At the beginning S_1 is left, S_4 open, S_5 down, S_6 up, S_7 left, and S_8 right. S_2 is closed and b adjusted to balance the potentiometer against the standard cell. Next S_5 is thrown up and r_0 adjusted to a balance. Then S_1 is thrown to the right and at the same time S_3 closed (opening S_2, these switches being mechanically interlocked so that both cannot be closed at one time), and the slide wire, m, is adjusted to a balance—reading E between the potential leads. The switches are then returned to their original positions, S_4 is closed and the same thing repeated. The change in the slide wire reading gives ΔE for constant filament heating current. S_8 is thrown to the left and S_7 to the right, and the potentiometer balance gives the voltage across the dial box and thus the value of i. In carrying out these measurements each slide-wire setting could be checked carefully in about two or three seconds when the conditions were satisfactory. Readings were taken against time before closing S_4 and again after closing S_4. Lines were passed through the resulting points and the interval between these lines used as ΔE.

C. Experimental Results.

The calorimetric measurement of φ requires the determination of the function

$$Q = \frac{2EI}{E - I\dfrac{dE}{dI}}$$

and of $\Delta E/i$. This latter involves the careful evaluation of the corrections to be applied to ΔE. Thus there are three sections of the work which will be taken up in order.

(Determination of Q.)

The only difficulty in connection with the determination of values of Q is in obtaining sufficiently accurate values of the differential coefficient

dE/dI. The procedure adopted was first to set up an empirical equation for E by evaluating the coefficients of the power series $E = A_0 + A_1 I + A_2 I^2 + A_3 I^3 + A_4 I^4$ and then to calculate dE/dI for the eight points from $dE/dI = A_1 + 2A_2 I + 3A_3 I^2 + 4A_4 I^3$. The labor involved in evaluating the coefficients in the equation for E was materially reduced by using pairs of values of E and I corresponding to equal intervals in I, and applying a method recently described by T. C. Tobin.[1]

The results of these measurements and calculations are collected in Table I. A considerably greater number of pairs of values of E and I

TABLE I.

$I / \frac{10.1870}{1.00145}$.	I (amp.).	E (volts).	E (Computed from Empirical Equation).	$\frac{dE}{dI}$ (volts/amp.).	Q (amp.).	Q (Filament) (Shunted).
.052.....	.52895	4.6757				
.055.....	.55947	5.1189				
.058.....	.58999	5.5821				
.061.....	.62050	6.0604	6.0604			
.064.....	.65102	6.5582	Exact			
.067.....	.68154	7.0715	7.0723			
Fixed res. No. 1 .	.70646	7.5026	—	17.580	2.1558	2.1508
.070.....	.71205	7.6033	Exact			
Fixed res. No. 2 .	.72182	7.7777	—	17.869	2.1927	2.1874
Fixed res. No. 3 .	.74054	8.1140	—	18.224	2.2330	2.2280
.073.....	.74257	8.1517	8.1518			
Fixed res. No. 4 .	.75773	8.4306	—	18.552	2.2707	2.2654
.076.....	.77309	8.7180	Exact			
Fixed res. No. 5 .	.78692	8.9802	—	19.106	2.3344	2.3287
.079.....	.80360	9.3023	9.3019			
Fixed res. No. 6 .	.80615	9.3520	—	19.467	2.3779	2.3718
Fixed res. No. 7 .	.82975	9.8157	—	19.901	2.4323	2.4258
.082.....	.83412	9.9220	Exact			
Fixed res. No. 8 .	.85151	10.2516	—	20.290	2.4852	2.4780
.085.....	.86464	10.5200	10.5211			
.088.....	.89515	11.1559	Exact			
.091.....	.92567	11.8088	—			

was taken than was actually required for the determination of values of Q. These were for use in connection with the temperature-emission measurements to be described later. The pairs of values designated as "exact" in column 4 are those used in evaluating the power series for E. The computed values of E in the same column agree with the observed values in column 3 almost within the limit of error of the measurements. The values of Q cannot be in error by more than a few parts in 10,000.

[1] Phil. Mag., 40, p. 513 (1920).

It should be noted that the above values of E refer to the filament when not shunted by the high-resistance bridge used for returning the space current to the filament ends, as in Fig. 1. When so shunted the current flowing through the filament is less than that measured on the potentiometer by the amount of the bridge current. The last column of Table I. gives the values of Q corresponding to the eight fixed resistance points with the filament shunted in this way. These are the values to be used in the case of space current returned to the ends of the filament or to the middle and ends.

The factor Q has the dimensions of current and is, therefore, independent of the length of the filament. When space current is flowing the heating current and also Q varies from point to point along the filament. Thus it is not strictly accurate to regard Q as a constant depending only on I. It can be shown, however, that for the case in which the space current is returned equally to the two ends of the filament, this variation is taken care of by multiplying the apparent value of φ by the factor

$$\left[1 - \frac{1}{12}\left(\frac{i}{Q}\frac{dQ}{dI}\right)^2\right].$$

Evaluating this by means of data from Tables I. and IV. it is found to differ from unity at most by only one part in 10^5.

(Determination of $\Delta E/i$.)

For all measurements in which space current is drawn from the filament it is necessary that the plate voltage be high enough to cause saturation of the emission over the main length of the filament and up to points near the ends where it is cut off fairly sharply by the boxes which shield the ends. It is impossible, of course, that the emission should be in voltage saturation quite up to the points of complete cutoff. An approximate approach to this condition is, however, possible by using a sufficiently high plate voltage. On the other hand, the correction to ΔE due to the back heating from the plates is proportional to the plate voltage and, therefore, this voltage should be as low as possible. From observations to be described later an effective plate potential of 196 volts, measured from the filament mid-point, was chosen as a satisfactory value. This effective voltage is the value read on the voltmeter decreased by the fall of potential caused by the space current flowing through the high-resistance bridge.

The procedure of measuring $\Delta E/i$ has been indicated in section B. Typical data giving one such measurement are shown in Table II.

TABLE II.

Measurement of $\Delta E/i$.

Fixed Res. Point No. 7—*i*Returned to Middle of Filament—Direction of Filament Current (1).

Time (Mins.).	Slide Wire Reading of E.	V (Volts).	Time. (Mins.).	Slide Wire Reading of E.	V (Volts).
0	.02696	Off	14	.04088	—
1	705	Off	14½	082	—
2	705	Off	15		Off
3½	691	Off	17	.02673	Off
4	695	Off	18	672	Off
4		On	19½	675	Off
6	.04095	204	20	672	Off
7	105	—	22½	690	Off
7½	096	—	23	686	Off
9½[1]		—	24	691	Off
11	094	—	25	692	Off
12½	079	—	26	690	Off
13	084	—			

By plotting carefully against time the values of the slide-wire reading of E we determine,

$$\Delta E = -.00287(0) \text{ volts.}$$

$$\text{Dial Box Res.} = 100.0 \text{ ohms;}$$

$$\therefore i = .006660 \text{ amp.}$$

$$\Delta E/i = -2.081 \text{ volts/amp.}$$

$$\text{Effective } V = 196 \text{ volts.}$$

At each of the eight fixed resistance points determinations of $\Delta E/i$ were made in this way for the space current returned to the middle of the filament, to the ends of the filament, and to the middle and ends. These measurements were made for the two directions of heating current through the filament, a total of forty-eight determinations of $\Delta E/i$. The results of this series of measurements at one typical point are shown in Table III. The product of $\Delta E/i$ with the corresponding value of Q is designated as φ'. In order to save space the results are not completely tabulated here.

TABLE III.

Fixed Res.	Direction of Filament Current.	Space Current Return.	$i \times 10^3$ (amp.).	$\frac{\Delta E}{i}$ (volts/amp.).	φ' (volts).	Mean φ' for 2 Directions of Fil. Current.
No. 7	1.....	Middle....	6.660	2.081	5.065	4.635
		Ends.....	6.351	2.222	5.395	5.207
		M. and E..	6.338	2.095	5.085	4.797
	2.....	Middle....	6.697	1.728	4.205	
		Ends.....	6.377	2.070	5.02	
		M. and E..	6.390	1.858	4.51	

[1] Pot. Reading on Dial Box, .66606.

It will be noted that the values of φ' for direction of heating current (1) are higher than for direction (2). (Throughout all the measurements the difference was uniformly about 14 per cent.) This failure of values to check on reversal results, as already mentioned, from geometrical dissymmetry. In the present case the failure was found to result from a slight difference in average diameter between the two halves of the filament, and from a dissymmetry in the location of the guard boxes. The effect to be expected from these departures from symmetry has been carefully estimated and found sufficient to account for the observed dependence of $\Delta E/i$ upon the direction of heating current. The mean values of φ' for the two directions are correct, so far as these effects are concerned, to the second order of small quantities.

These mean values of φ' are plotted against i in Fig. 4. The points

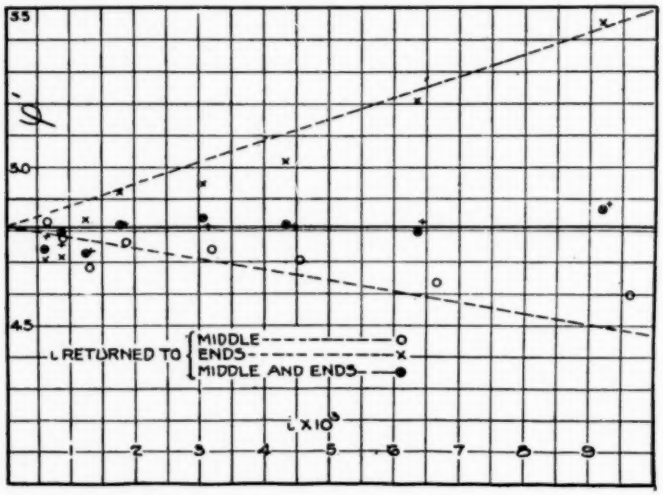

Fig. 4.

marked + are placed 1/3 the distance from the ⊙ points to the × points. Note that these coincide almost perfectly with the ⊕ points as required by theory,—except for values of i below 1 mil where ΔE is very small and the experimental errors in determining the values of φ' are large. Taking 4.81 to be the true mean value of φ', the temperature shift formulæ of section *A* lead to the curves represented by the dotted lines. These are in reasonably good agreement with the observed points. We are thus justified in concluding that the temperature shift effect is eliminated from the values of φ' for the space current returned to the middle and ends.

(Corrections to φ'.)

In Fig. 5 the mean values of φ' of Fig. 4 are replotted against abscissæ representing absolute temperatures. (The determination of these

temperatures is taken up in Part II.) Only the ⊕ points and the + points are shown in Fig. 5. Each point, if it were corrected for the back heating from the plates and for the effect of the electrostatic shields, would represent an experimental measurement of φ itself. The actual method employed in the final evaluation of φ consisted of a careful determination of these two errors at the fixed resistance point No. 7, and an application of the results to the best value of φ' obtained by smoothing the points of Fig. 5.

The solid line in Fig. 5 is drawn for $\varphi' =$ constant and the dotted line represents

$$\varphi' = \varphi_0' + \frac{3}{2}\frac{kT}{e},$$

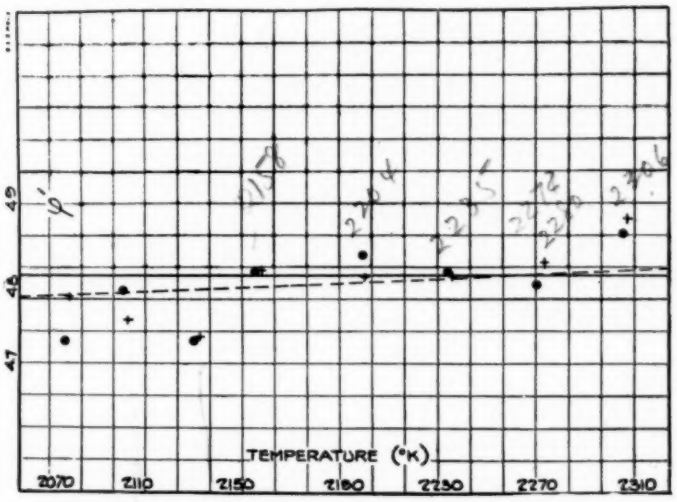

Fig. 5.

corresponding respectively to the first and second forms of Richardson's equation. Neither of these lines seems to fit the points as well as they can be fitted. The best line would have a slope about twice that of the dotted line, *i.e.*, about $3k/e$. It is possible, however, that this greater variation is more apparent than real. Reasons are given in another section for thinking that the value found for φ' at 2306° K. is higher than the true value for this temperature, while the values found at the lowest temperatures are subject to considerable error on account of the smallness of the emission. On the other hand, it does not seem impossible to modify the thermodynamic requirements in this connection by modifying somewhat our view in regard to the nature of the system. We hope to consider this possibility more fully in a paper now in preparation on the work function of oxide-coated filament.

Whatever the variation of the work function with temperature, Fig. 5 determines the value of φ' at the fixed resistance point No. 7 to be 4.81 volts, which is probably correct to $\pm$ .015 volt. This is the final value of φ'. Correction must be made for the effect of the electrostatic shields and for the back heating from the plates before the true value of φ is obtained.

The error due to the electrostatic shields results from the fact that if the two shields are maintained at the same negative potential relative to the middle point of the filament, as was the case in the measurements so far described, the "grid action" at the positive end of the filament is more marked than at the negative end. This has the effect of shifting the center of emission toward the negative end of the filament and of introducing an error in ΔE which does not reverse sign relative to the true effect on reversal of the heating current. It is possible to avoid this error by maintaining the same difference of potential between shield and adjacent portion of filament at each end. The importance of this correction was not appreciated, however, until observations were nearly completed, and the shields were maintained throughout at $-$ 11.57 volts relative to the middle point of the filament.

It can be shown that if the center of emission shifts a distance δ when the heating current is reversed a spurious voltage change is introduced which is given by

$$\Delta E'' = -\frac{i\delta}{2L}\frac{dE}{dI},$$

regardless of the way in which the space current is returned to the filament. This results, if not corrected for, in a value of φ' which is in error by being too large.

The magnitude of δ for the seventh point has been calculated from the change in emission which occurs when the potential of the shield at the positive end of the filament is increased from its normal value by an amount equal to the potential drop along the filament between the shields. This makes the potential difference between filament and shield the same at the two ends, and causes an increase in current corresponding to δ = .0335 cm. for one direction of heating current and to δ = .0352 cm. for the other. The difference of about 5 per cent. between these values probably arises from a slight geometrical difference between the situations of the boxes about the filament. The mean of these corresponds to a correction to φ' of $-$ 2.24 per cent.

To check this calculated correction, values of φ' were redetermined for the two directions of heating current, maintaining the proper relations

between filament and box potentials. Successive measurements gave values lower than the former values by respectively 1.50 per cent. and 2.01 per cent. for one direction of heating current and 2.52 per cent. and 1.74 per cent. for the other direction. These are in agreement with the computed amount of the correction, although the accuracy of the direct measurements is not sufficiently good to afford a close quantitative check.

The original value of φ' corrected by $-$ 2.24 per cent. gives 4.702 volts, and this we regard as more reliable than the redetermined values. There remains only the correction due to radiation from the plates.

The method of determining the correction due to the heating of the plates has been indicated in section A. It consists in plotting $\Delta E/i$ against plate voltage for constant heating current with i returned to the middle and ends. This was carried out at the 7th point of the fixed resistance, for direction No. 1 of the heating current and with the boxes maintained in the correct potential relations to the main filament. The procedure adopted in these measurements was to determine

$$\delta\left(\frac{\Delta E}{i}\right) = \frac{i\delta\Delta E - \Delta E\delta i}{i^2}$$

for changes in plate potential from the standard value, 196 volts, to other potentials above and below this value. The data were then used to evaluate the ratio

$$\delta\left(\frac{\Delta E}{i}\right)\Big/\left(\frac{\Delta E}{i}\right)_{196}.$$

The results of these measurements are shown graphically by the points in Fig. 6. It will be noted that from $V = 140$ volts to $V = 350$

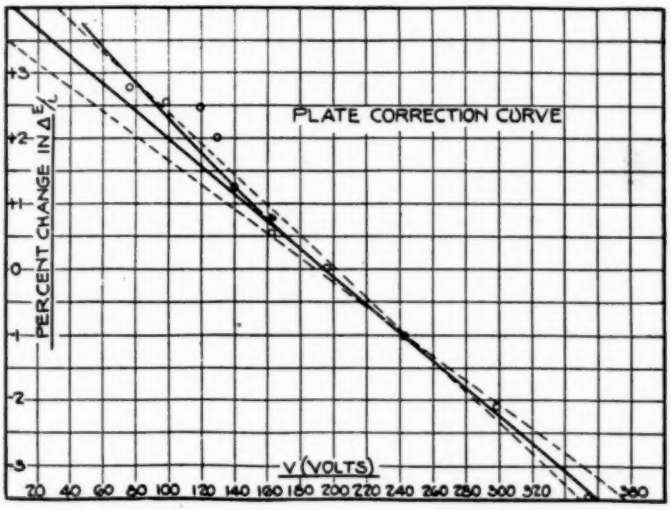

Fig. 6.

volts the variation in $\Delta E/i$ is sensibly linear. The best line through these points has a slope $-$ 0.0210 per cent. per volt, so that the apparent value of φ for $V = 196$ volts must be increased by $196 \times 0.0210 = 4.12$ per cent., this fraction applying not to the above corrected value of φ' but to the value which would be actually measured under the conditions of the test. This value is 5.036 volts, and thus the value of φ for the seventh point is increased from 4.702 volts to the final value 4.91 volts. The estimated possible errors in this plate correction determination are represented in the figure by the two dotted lines.

In establishing lines in Fig. 6 the points for V less than 140 volts have been disregarded because of another voltage dissymmetry which gives rise to spurious effects when the plate potential is sufficiently low. At each end of the filament the emission is cut off more or less sharply near the point at which the filament enters the guard box. The position of this cut off depends not only upon the difference of potential between box and filament, but also upon that between plate and filament and upon the density of emission. If the plate potential is sufficiently high the fact that the difference between plate and filament is greater at one end than at the other is unimportant. As the plate potential is reduced, however, this dissymmetry becomes more marked and results in an appreciable shift of the center of emission toward the negative end of the filament, causing a spurious increase in $\Delta E/i$. It was from preliminary measurements of the plate correction at the seventh point that 196 volts was chosen as sufficiently high to make the effect of this shift negligible. We believe the measured values of φ' at the eighth point are high (see Figs. 4 and 5) because this voltage was too low for the density of emission from the filament at this point.

As a check on the plate correction we may calculate roughly what correction is to be expected. It was shown in another section that the correction to φ is given by $-fV$, where f is the fraction of the power dissipated at the plates that is absorbed by the filament. The diameter of the filament is .00755 cm. and its distance from the plates 0.63 cm. If we assume that the power is dissipated along the normal projections of the filament on the plates and that the radiation proceeds from these lines, equally from the two sides of each plate and in accordance with the cosine law, then the fraction .003 of the total radiation is intercepted by the filament. If the coefficient of absorption of the filament for this radiation is λ, then $f = .003\lambda = \Delta\varphi/V$. From the measurements $\Delta\varphi/V = 0.04\varphi/196$. Taking φ as 5 we find $\lambda = 0.35$ which is a reasonable value for this constant. We conclude, therefore, that the experimental estimate of the plate correction is about what might be expected.

We have thus applied two corrections to the original apparent value 4.81 volts and have arrived at 4.91 volts as the final correct value of φ at the seventh point. The temperature for this point is 2270° K.

The possible absolute errors in determining, first, the apparent value of φ, second, the voltage-shift correction, and third, the plate correction are estimated to be $\pm$ .015 volt, $\pm$ .005 volt, and $\pm$ .025 volt. The most unfavorable cumulative error we thus estimate as $\pm$ 0.045 volt or about one per cent. of the value of φ.

PART II. MEASUREMENT OF "b" IN RICHARDSON'S EQUATION.

The more common form of Richardson's equation is

$$i = AT^{1/2}\epsilon^{-b/T}.$$

The constants of this equation, A and b, can be readily determined and the adequacy of the equation tested by plotting ($\log i - \frac{1}{2}\log T$) against $1/T$. This should give a straight line having the slope $-b \log \epsilon$. From the value of b obtained in this way, φ, the equivalent voltage of the work function, is determined from the relation, $\varphi = bk/e$. It is the purpose of this part of the paper to describe the determination of φ in this way, and to compare the value with that obtained by the direct calorimetric measurement.

With the potentiometer arrangement described in Part I. the relation between the emission from our filament, i, and the heating current, I, was readily determined with great accuracy. Emission measurements were made over as wide a temperature range as possible, with the effective plate potential held at 196 volts. Both of the limits of this range were probably determined by gas in the tube, the emission being extremely steady except at very low and very high temperatures. For high temperatures the power dissipated upon the plates liberated sufficient gas from them to seriously change the emission. At very low filament temperatures gas was probably absorbed by the filament, resulting in a decrease in the emission.

Over the range from 0.1 to 10 milliamperes the emission was always extremely steady and data were taken only for this range. With the effective plate potential maintained constant, as well as the filament heating current, there was no certain difference in emission for the three methods of returning i to the filament. Also there was no change of i on reversal of the heating current. The emission data, upon which the determinations of b are based, are given in columns (1) and (2) of Table IV. The starred values of I correspond to the eight fixed resistance points used in the calorimetric determination. *It is of importance to*

note that the emission data were taken in the midst of the ΔE determinations, and that the condition of the filament was the same during these two sets of measurements.

TABLE IV.

I (amp.).	$i \times 10^3$ (amp.).	Temperature (°K.).		I (amp.).	$i \times 10^3$ (amp.).	Temperature (°K.).	
		Langmuir.	W. and F.			Langmuir.	W. and F.
.52895		1772	1779	.75773*	1.902	2138	2158.0
.55947		1823	1833	.77309	2.538	2162	2182.0
.58999		1874	1884	.78692*	3.269	2183	2204.0
.62050	.09341	1923	1935.5	.80360	4.405	2207	2231.0
.65102	.1973	1973	1986.5	.80615*	4.606	2212	2235.0
.68154	.3967	2021	2036.0	.82975*	6.875	2247	2271.5
.70646*	.6784	2060	2077.5	.83412	7.394	2254	2280.0
.71205	.7656	2069	2086.5	.85151*	9.792	2279	2306.0
.72182*	.9363	2084	2102.0	.86464		2298	2327
.74054*	1.362	2113	2131.5	.89515		2342	2375
.74257	1.419	2116	2134.5	.92567		2386	2420

The accuracy of the measurement of b depends entirely upon the determination of temperature as a function of I. The temperatures were found from data for tungsten published by Langmuir,[1] and by Worthing and Forsythe,[2] using our measured values of E and I and the diameter and length of the filament.

The temperature of a filament is uniquely determined by its resistivity, $\pi d^2E/4lI$, or by the power radiated per unit area, $EI/\pi dl$. Here d is the diameter of the filament and l its length. For the sake of simplicity Langmuir used the functions, d^2E/lI and EI/dl. The temperature is, of course, also completely determined by the function resulting from any combination of these two functions. By eliminating successively each of the variables, I, d and l, we can obtain the three other "temperature functions," $(E\sqrt{d})/l$; $(E\sqrt[3]{I})/l$; $I/d^{3/2}$. Langmuir published values of these five functions for tungsten at temperatures from 300° K. to 3540° K., at steps of 100°. We plotted these tabulated values carefully on accurate cross-section paper. For each value of I in Table IV. the temperature was found in five ways by locating the directly measured temperature functions upon these curves.

The data of Worthing and Forsythe are contained in the two empirical equations,

$$\log_{10}\left(\frac{EI}{\pi dl}\right) = 3.680\ (\log_{10} T - 3.3) - \frac{1040}{T} + 1.900$$

[1] PHYSICAL REVIEW, 7, p. 315 (1916).

[2] PHYSICAL REVIEW, 18, p. 144 (1921).

$$\frac{\pi d^2 E}{4lI} = 59.2 \times 10^{-6} \times \left(\frac{T}{2000}\right)^{1.200}.$$

In the second of these equations 59.2×10^{-6} represents the resistivity of tungsten at 2000° K. From these equations the five temperature functions given above were computed at steps of 25° over the range covered by the emission data. From the resulting curves five different determinations of temperature were obtained for each value of I, in exactly the same way as from the curves of Langmuir's figures. Temperatures could be read to within nearly 1° from Langmuir's curves and to within 0.5° from the curves of Worthing and Forsythe.

The temperatures actually determined from the five temperature functions differed considerably at each value of the heating current, for both the scale of Langmuir and the scale of Worthing and Forsythe. The only possible explanations of this discrepancy are, either, (1) our filament differs from the samples of tungsten forming the bases of the temperature scales; or (2) our measured values of the temperature functions are incorrect.

If this latter is the true explanation it can be easily shown that the error is entirely confined to our value of d. E and I are known with extreme accuracy and l with fair accuracy, while the measured diameter, 7.595×10^{-3} cm., is less accurate, and it is probable that this value was somewhat decreased by vaporization during pumping.

In examining for such a possible error temperatures were computed, at a given value of the heating current, for a number of assumed values of d. Plotting computed temperatures against d the curves corresponding to the different temperature functions intersect at a value about 0.5 per cent. lower than the measured diameter. Curves of this kind, based upon the scale of Worthing and Forsythe, were plotted for four different heating currents, including the two extreme values of the $E - I$ curve. At each heating current the curves intersected at the same value of d to well within the limit of experimental error in determining the curves. The actually measured points of intersection were as follows:

I (amp.)	.52895	.68154	.80360	.92568
d at intersection (cm. $\times 10^3$)	7.550	7.554	7.556	7.553

Using the mean value $d = 7.553 \times 10^{-3}$ cm., all of the five temperature functions determine the same temperature to within 1° at each of the twenty-two points of the $E - I$ curves. This is a temperature range of 641° K.

The fact that values of d determined for different values of I differ at most by one part in 1000 is an indication that the tungsten of our filament

is the same as that for which Worthing and Forsythe's equations have been written. It is not, however, a proof of such identity since the constants of these equations can be altered somewhat and an equally good series of values of d obtained. This point has been carefully examined in connection with the variations in the constants from one sample of pure tungsten to another as reported in connection with the equations. As a result of this examination we believe that the temperatures which have been associated with the heating currents do not differ from Worthing and Forsythe's scale by more than two or three degrees. As regards true temperatures we carry over, of course, whatever differences there may be between Worthing and Forsythe's scale and the thermodynamic scale.

Temperature determinations from Langmuir's figures were not so consistent. For heating currents over most of the $E - I$ curve this scale yielded values of the diameter in the range 7.550 to 7.555×10^{-3} cm., in exact agreement with the other determinations. For low currents, however, a smaller value of the diameter was determined, the lowest point of the $E - I$ curve giving $d = 7.51 \times 10^{-3}$ cm. The best temperature determinations it was possible to obtain from Langmuir's data are based on the function, $(E\sqrt{I})/l$. Taking $d = 7.55(3) \times 10^{-3}$ cm., these temperatures are in good agreement with the determinations from the other functions except for quite low values of I. At the lowest point of the $E - I$ curve the greatest difference is 14° K.

The temperature scale of Worthing and Forsythe is probably more accurate than that of Langmuir. Furthermore, our measurements do not give consistent results on the latter scale. Consequently the series of temperatures found from the scale of Worthing and Forsythe should be given much more weight than the temperatures from Langmuir's scale. These sets of values are given in columns (4) and (5) of Table IV.

The value of "b" was obtained in the ordinary way by plotting $(\log_{10} i - \frac{1}{2} \log_{10} T)$ against $(1/T)$. For the highest points the temperatures have been corrected by the computed temperature change arising from the cooling, the greatest correction amounting to -3.1 degrees. The plotting was done with care on large cross-section paper. The points fall upon two straight lines, corresponding to the two sets of temperatures, with an accuracy considerably greater than could be shown in a figure here. No one of the points on the basis of Worthing and Forsythe's scale is more than one degree from its line. The maximum departure for the points based on Langmuir's scale is 2.3°.

The slopes of these two lines divided by $-\log_{10} \epsilon$ give the following values of b:

	b (degrees K.).	φ (volts).
Scale of W. and F.	5.541×10^4	4.778
Scale of Langmuir	5.646×10^4	4.869

In calculating corresponding values of φ we have used $k = 1.372 \times 10^{-16}$ ergs per degree and $e = 4.774 \times 10^{-10}$ e.s.u.

Of the two values of φ every consideration requires that we put greater reliance in the former. Some idea as to the probable error of this value may be gained from the consideration that, if the temperature at the lowest point is correct and the value of φ is one per cent. in error, then the temperature at the highest point must be in error by 4.3 degrees. On the other hand a uniform error of 10 degrees from one end of the range to the other produces only a one per cent. change in φ. It seems unlikely, then, that this value of φ should be in error by much over one per cent.

Discussion of Results.

The value found for φ by the calorimetric method, 4.91 volts, is 2.7 per cent. greater than the value, 4.778 volts, found for the same constant from temperature-emission data on the basis of Richardson's equation. We believe that this difference is greater than can be reasonably accounted for by experimental error. It was pointed out in the introduction that a difference between these values might exist because of the impossibility of properly inferring the pressure of electrons in equilibrium with a metal from the rate of emission of electrons from it. If this is the cause of the difference, the calorimetric value is to be regarded as the true value, and that determined from the temperature-emission relation as an apparent value only. Before accepting this explanation as likely it will be well to consider again just what the quantities are that are being compared.

In previous measurements of the calorimetric φ by the Wheatstone bridge method it has been customary to subtract from the total energy absorbed per electron the quantity $2k(T - T_0)$. This has been done on the assumption that electrons entering the bridge arm at temperature T_0 absorb this quantity of energy, on the average, before leaving the filament at temperature T. This term is $2k(T - T_0)$ rather than $3/2k(T - T_0)$ because we are dealing with electrons passing through surfaces rather than electrons contained in given volumes. On this basis our calorimetric value of φ requires no such correction since all measurements have been made on a filament of uniform temperature. The energy absorbed per electron measures directly the work done against electrical forces.

To be consistent with this view in regard to the calorimetric φ, we must suppose that the φ in Richardson's equation is also the equivalent voltage of work done against electrical forces only. Richardson has shown by means of a thermodynamic argument that, in general, φ must vary with temperature, and that to a close approximation it will depend upon temperature through the relation

$$\varphi = \varphi_0 + \frac{3}{2}\frac{k}{e}T,$$

where φ_0 is a constant. The approximation is in the coefficient of T which will differ slightly from $(3/2)(k/e)$ by an amount which depends for a particular metal upon the coefficient of its Thomson effect. If we disregard the approximate nature of this relation the general equation

$$i = AT^{1/2}\epsilon^{\int_T \frac{\varphi e}{kT^2}\,dT}$$

reduces to

$$i = AT^2\epsilon^{-\frac{\varphi_0 e}{kT}}.$$

The proper procedure in determining φ for temperature T is to determine φ_0 from the "T^2 equation" and then to add to this the quantity $(3/2)(k/e)T$. If instead, φ is evaluated from the "$T^{1/2}$ equation" what one obtains is an average value of $(\varphi_0 + (3/2)(k/e)T)$ for the range of the measurements. The difference between these quantities in the present measurements is comparable with the experimental error in determining the slopes of the straight lines. It seems, therefore, that the measurements have been made in accordance with the requirements of the classical theory, and that a better agreement was to have been expected than has been obtained.

It is generally recognized, however, that while the classical theory affords a satisfactory explanation of thermionic emission, and has one or two other notable achievements to its credit, it affords on the whole an unsatisfactory account of the electrical and thermal properties of metals. We should consider, therefore, whether a different theory may be expected to lead to a different set of thermionic relationships. This question has been considered by Richardson[1] who finds that differences are to be expected, and these, it appears, may very well be of a sort to account for the discrepancy we find between the differently determined values of φ. If we assume, for example, that the conduction electrons in a metal possess no thermal energy whatever, then the heat absorbed per electron leaving the surface of a filament is a measure of $\varphi e + 2kT$,

[1] O. W. Richardson, "The Emission of Electricity from Hot Bodies," 1st ed., p. 169; 2d ed., p. 185.

the work done against electrical forces plus the thermal energy which the emitted electron is known from direct experiment to possess.

Consistent with this we must suppose, in the derivation of Richardson's equation, that when a system consisting of an enclosure containing a body in equilibrium with its electron atmosphere at temperature T undergoes an isothermal expansion v, the heat absorbed is $nv(\varphi e + (3/2)kT) + pv$, instead of the same expression with the $(3/2)kT$ term omitted. This term does not appear on the classical theory because the average thermal energy of the electrons is supposed the same inside as outside the metal, ($(3/2)kT$ in each case). The difference that this makes in the interpretation of Richardson's "$T^{1/2}$ equation" is that the measured value of φ, which we will now refer to as φ_1, is no longer the equivalent of work done against electrical forces but that $\varphi_1 = \varphi + (3/2)(k/e)T$, where φ is this equivalent. This gives at once a reasonable explanation of the required variation of φ_1 with temperature, that is, of the relation

$$\varphi_1 = \varphi_0 + \frac{3}{2}\frac{k}{e}T.$$

On the present theory φ_0 is identified with φ, and this term alone represents work done against electrical forces. The second term is, in a sense, a proper part of the work function, but it does not represent a linear increase in the restraining voltage with temperature. Another alteration is that the approximation in the expression for φ_1 is no longer confined to the second term, so that, φ_0 (or φ) may vary somewhat with temperature.

What this means in connection with the present experimental results is that the equivalent voltage of work done against electrical forces alone is obtained by subtracting $2(k/e)T$ from the calorimetric value of φ, or $(3/2)(k/e)T$ from the value calculated from the temperature-emission relation. Making these subtractions for $T = 2270°$ K., we have

$$\varphi = 4.91 - 0.391 = 4.52 \text{ volts (calorimetric)}$$
$$\varphi = 4.778 - 0.293 = 4.48 \text{ volts (temperature-emission)}$$

i.e., values of φ are obtained differing by less than one per cent.

While it may be reading too much into the results to say that they differentiate between the two theories in favor of the view that the conduction electrons in metals possess little or no thermal energy, it may at least be said that they are more consistent with this view than with the classical theory.

It may be cited as further evidence in support of this view that our value of φ by the calorimetric method is in better agreement with that

of H. H. Lester on the non-classical than on the classical theory. Lester's value, 4.478 volts, was obtained by subtracting $2k/e(T - T_0)$ from the total equivalent of the energy absorbed per electron. This should be the correct value on the classical theory. On the non-classical theory the quantity subtracted would have been $2k/eT$, giving $\varphi = 4.438$ volts. The agreement between our value of φ and Lester's is thus better on the latter theory. This will be clearer perhaps if all of the figures are set down together.

	Lester.	D. & G.	Diff.
Classical theory	$\varphi = 4.478$	4.91	9.6%
Non-classical theory	$\varphi = 4.438$	4.52	1.9%

There is some difficulty, of course, in understanding how it is that, if the conduction electrons in a metal do not possess normal thermal energy, they nevertheless escape from the surface with the average energy $2kT$. The way out of this difficulty may be contained in Borelius'[1] suggestion that while the conduction electrons in the body of a metal possess only a small fraction of normal thermal energy those near the surface possess the normal amount.

In connection with some more recent work we have been led to the consideration of quite a different possible explanation of the difference between the values of φ found by the two methods. As this cannot well be taken up here it will be presented in a later publication.

We take pleasure in expressing our thanks to Mr. R. E. A. Putnam of this laboratory who contributed much in preliminary experiments to the development of the present calorimetric method, and to Drs. H. D. Arnold and W. Wilson for the interest they have taken in this work, and for the encouragement they have given us.

RESEARCH LABORATORIES OF THE
AMERICAN TELEPHONE AND TELEGRAPH COMPANY,
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Feb. 1, 1922.

[1] Phil. Mag., 40, p. 746 (1920).

THE RELATIVE SENSITIVITY OF THE EAR AT DIFFERENT LEVELS OF LOUDNESS.

BY DONALD MACKENZIE.

SYNOPSIS.

Alternation Phonometer for Balancing the Loudness of Tones of Different Pitch.—It has been found that if two tones alternate in the ear at a suitable rate, for instance 25 times a second, the interruptions of the louder tone are the more conspicuous, and it is possible to adjust the relative intensities until the interruptions are equally conspicuous. Two tones when balanced in this way are said to be equally loud. This definition is justified by the fact that if two tones each balance a third tone they are found to balance each other and are also accepted by the ear as equally loud. The electrical circuit, commutating relays, and other apparatus used in making a loudness balance are described. To enable the mean pressure of the sound at the ear drum to be calculated from the electrical energy input, either a thermophone, whose theory is known, or an electromagnetic receiver which had been calibrated (rather imperfectly) by means of an artificial ear cavity and an electrostatic transmitter, was used next the ear. Successive readings usually agree within 5 per cent.

Relative Sensitivity and Sensibility of the Ear, at Various Loudness Levels, for Frequencies of from 100 to 4,000 per Second.—(1) Phonometric comparisons with a thermal receiver were made by ten men and ten women over a limited range of loudness. The results correspond to the equation $\log P_1 = A + B \log P_2$, where P_1 and P_2 are the acoustic pressures, computed from the thermophone theory, of the two balanced tones. This equation is in harmony with Fechner's law. The constant $-A$ is the sensitivity and depends only on the threshold pressures; $1/B$ is the relative sensibility at the two frequencies and is found to be constant, within 10 per cent., for the entire range of frequencies investigated. The curves for men and for women show no systematic differences except at the higher frequencies where resonance of the ear cavity begins to enter. (2) With an electromagnetic receiver a greater loudness range was possible, the pressure being varied up to 20,000 times the threshold values. Of 32 curves obtained with four observers making comparisons against 700 cycles, 25 were straight throughout, in agreement with the thermophone results, and only 7 were distinctly curved near the extremes. The slope at all frequencies is between 0.9 and 1.0; therefore the sensibility is practically uniform, at least to 2,000 cycles and at all loudness levels. No evidence of a phonometric Purkinje effect was observed, but it may be present for pressures near the threshold values. Since the pressures are all computed, the absolute values of A probably contain small systematic errors, but these would not affect the values of B.

INVESTIGATIONS of the sensitivity of the ear at the threshold of audition enjoy the advantage of dealing with one frequency at a time. If it is desired to study ear sensitivity at higher levels, means must be devised for the adjustment of different frequencies to equality of loudness. This done, data may be obtained showing physical intensities of sounds which to the ear are equally loud.

METHODS OF SOUND COMPARISONS.

The customary method of making loudness balances is to listen to o 1

frequency, then to the other, changing the intensities until the sounds are accepted as equally loud. This measurement can be made with some precision when the sound sources compared are emitting the same note, but is highly uncertain with two notes of different pitches. Aside from the flagging of the attention, there occurs a decay of the sensation,—the mechanism which appreciates loudness is fatigued as the sound continues. Presumably, a different mechanism responds to the second frequency, and the comparison made is between a decayed response to f_1 and a fresh response to f_2. This decay of sensation is readily observed by listening to a steady tone which suffers from time to time interruption for a perceptible interval,—after each interruption the sound seems louder, although the physical intensity has not altered.

It might be affirmed that such an error is offset by the opposite error when the order of the compared frequencies is reversed. In point of fact, however, each change leaves the ear in doubt, and the continuance of the work results in fatigue of all the auditory mechanisms involved. The net result is confusion and the impossibility of obtaining consistent observations.

In optical work, the flicker photometer has made it possible to compare the brightnesses of different colors. A flicker frequency is obtainable such that the colors fuse subjectively, and fluctuations in intensity, as well as the distinction of color, disappear at a definite brightness ratio of the colors compared.

A mechanically operated device, termed a flicker phonometer, has been patented by Heck.[1] The essential feature of this invention, as disclosed in the patent, is a sound-insulated valve automatically rotated at adjustable speed. Two tubes separately receive the sounds it is desired to compare, and each tube conveys an acoustic wave to the rotating valve, by which one tube is opened as the other is closed. Through the valve the tubes discharge sound to horns movable on a track, each facing the inlet to a tube leading to the ear. The claim is made that by proper disposition of the horns and regulation of the valve, pulsations in intensity of the sound reaching the ear can be caused to disappear. It is not apparent that the inventor contemplated the comparison of sounds of widely different pitches.

In this laboratory it has been found possible by electrical means to make use of the perception of flicker in an alternation phonometer. Let two frequencies alternate in the ear at a suitable rate, the intensity of one being maintained constant while the other is varied from faint to loud in comparison. The strong tone is heard plainly interrupted, ac-

[1] Method and Apparatus for Comparing Sounds, C. M. Heck: U. S. Pat. No. 1,356,359, October 19, 1920.

companied by the weaker tone whose interruptions are not clearly distinguished. If now the weak tone is progressively strengthened, its interruptions begin to claim the observer's attention and with further increase in intensity come to overshadow the constant tone just as they were previously overshadowed when faint. A little practice enables the observer to determine a point at which the ear recognizes the two tones as equally interrupted: the attention is here impartially divided, whereas on either side of this point one or the other of the tones makes the stronger impression. Such a point may be identified with a loudness balance.

This statement is justified by the following considerations: (1) unpracticed observers can reproduce their balances within 4 per cent.; (2) on comparing in the usual manner the tones so equated, it is not possible to affirm which is the louder; (3) if in this way f_1 is compared with f_2, and the value of f_2 just found is balanced in turn against f_3, then f_1 and f_3 have values which on direct comparison are found to balance.

Comparisons of this kind avoid difficulties due to the shortness of the persistence of audition and to ear-fatigue. It is, moreover, plausible to urge that the level of any sensation is specified by the attention paid to the stimulus, and it is on this basis that the alternation phonometer is operated.

THE ALTERNATION PHONOMETER.

The currents furnished by two oscillators are alternately impressed on an electrical generator of sound by the armature of a polarized telegraph relay, driven from another such relay at a controllable speed. The relays and their operating circuit are shown in Fig. 1. The use of these relays and the circuit of Fig. 1 were suggested by Dr. W. S. Gorton of this laboratory.

Adjustment is carefully made to secure (within 1 per cent.) equality of duration of contacts on the two sides, and travel time between contacts not greater than 0.002 second. This travel time should not be exceeded, it being desirable to avoid intervals of silence. Within this limit, available sounding intervals range from 0.020 to 0.080 second. An interval of 0.040 second is suitable for frequencies 100 to 4000.

The currents commutated by the driven relay are amplified and supplied either to a gold-leaf thermophone or to an electromagnetic receiver. Fig. 2 is a diagram of the circuit between the relay commutator and the sound generators.

SPECIMEN RESULTS.

In preliminary experiments the procedure was to keep fixed the intensity of one of the tones and vary the other until a balance was reached. Comparisons with the thermophone were made by one observer of a large

number of frequencies from 100 to 4,000 cycles. The r.m.s. alternating pressures in the cavity consisting of the thermophone case and the ear canal were computed from a theoretical formula developed by Dr. E. C. Wente.[1]

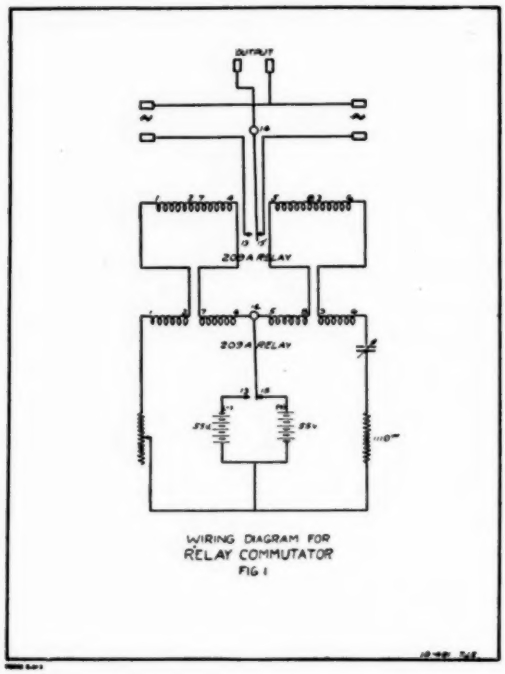

Fig. 1.

If we compare 400 vs. 500, 500 vs. 600, we can infer the result of a direct comparison of 400 vs. 600. Call p_1 the r.m.s. alternating pressure due to a frequency f_1 balanced by p_2 due to f_2, p_3 due to f_3. For the case where $f_1 = 400$, $f_2 = 500$, $f_3 = 600$, we find the following results:

TABLE I.

Observer: T. J. E.

Fixed in Volume.		Varied in Volume.	Varied / Fixed ; Ratio of Pressures at Balance.
400 (f_1)	*vs.*	500	0.76
400	*vs.*	600	0.66
500 (f_2)	*vs.*	400	1.33
500	*vs.*	600	0.84
600 (f_3)	*vs.*	400	1.70
600	*vs.*	500	1.22

Here

$$\frac{p_1}{p_2} = 1.33; \qquad \frac{p_2}{p_3} = 1.22; \qquad \therefore \frac{p_1}{p_3} = 1.62.$$

[1] "The Thermophone," PHYS. REV., 19, pp. 333–345, April, 1922.

By direct comparison,

$$\frac{p_1}{p_3} = 1.70.$$

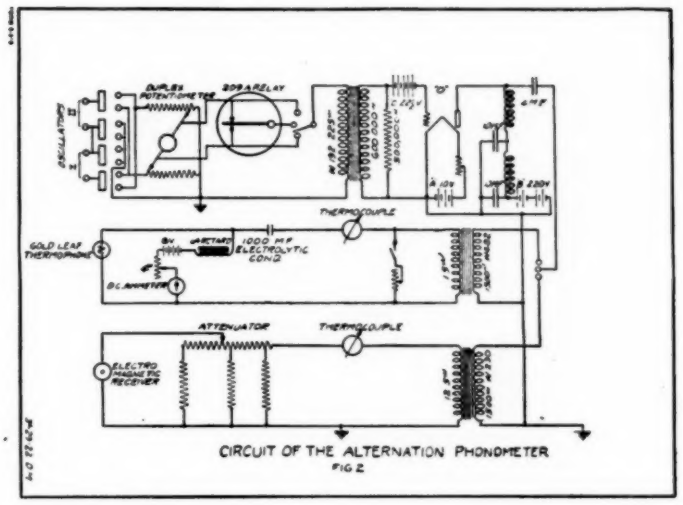

Fig. 2.

Again, when the frequencies exchange rôles, the one previously maintained at fixed volume becoming the variable, we find

$$\frac{p_2}{p_1} = 0.76; \qquad \frac{p_3}{p_2} = 0.84; \qquad \therefore \frac{p_3}{p_1} = 0.64.$$

By direct comparison,

$$\frac{p_3}{p_1} = 0.66.$$

The question arises, does the pressure-ratio at balance vary with the absolute sensation level? To answer this, comparisons of 100 vs. 300 were made at pressures ranging from 1 to 8 dynes/cm.^{2} for 300, 2 to 26 dynes/cm.^{2} for 100. Figs. 3 and 4 are curves representing these data, obtained by two observers.

The data of one observer are plotted in the upper curve of Fig. 3. The lower curve of this figure represents data taken by the same observer, comparing the same frequencies, on the following day. In the second case, a slower rate of commutation was used, the higher speed being judged too high for satisfactory perception of the 100-cycle tone.

The computed points are obtained from a law suggested by a logarithmic plot. It is seen that there is good agreement between the points so computed and those actually observed.

CONCORDANCE WITH FECHNER'S LAW.

Ab initio, we should feel no surprise at finding a logarithmic law. If we are working at a sensation level within the range of Fechner's law, the

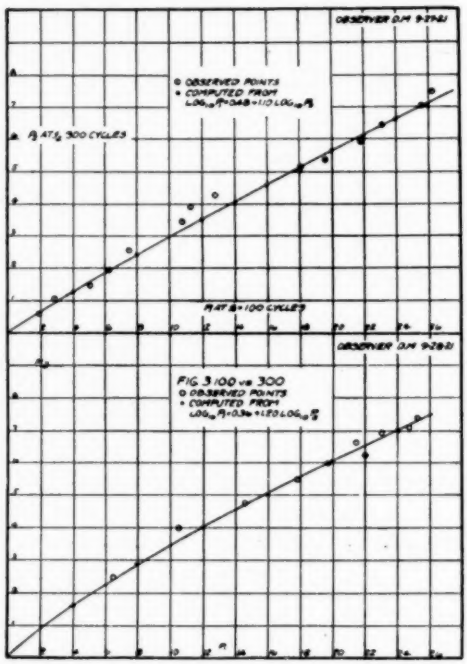

Fig. 3.

sensation should be connected with the stimulus (in this case loudness with pressure) by the relation

$$S = c \log P + a.$$

When two frequencies are at the same loudness,

$$S = c_1 \log P_1 + a_1 = c_2 \log P_2 + a_2$$

or

$$\log P_1 = \frac{a_2 - a_1}{c_1} + \frac{c_2}{c_1} \log P_2.$$

Within the range of validity of such a law, then, we should expect the balances to obey a law of the form

$$\log P_1 = A + B \log P_2.$$

Further evidence confirmed this expectation. Fig. 4 shows logarithmic plots of the data of another observer (T.J.E.) comparing 100 *vs.* 200, 200 *vs.* 300, 100 *vs.* 300. The logarithmic law satisfies observations on frequencies from 100 to 4,000.

At the higher frequencies (2,000 to 4,000) difficulties were found in checking indirect with direct comparisons. If the thermophone case is not held tightly against the ear, a small opening converts the cavity into a Helmholtz resonator and so enhances the effect of the tone whose frequency is the nearer to the natural frequency of the resonating cavity. This source of error may be precluded by care in manipulation. An additional uncertainty is introduced by harmonics contaminating the tones,—this difficulty is remedied by the introduction of filters. Failure to check direct against indirect comparisons may also be due to omitting the precaution to make comparisons at a fixed sensation level.

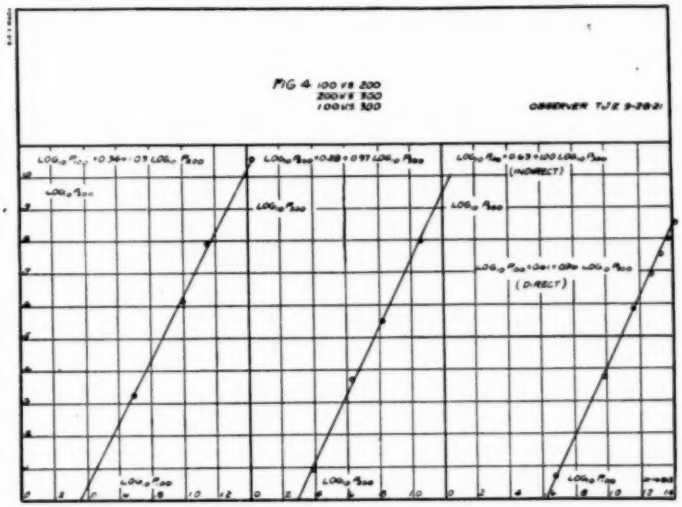

Fig. 4.

When care is taken to conform to these three specifications (tight fit of thermophone against ear, purity of tones compared, fixed level of sensation), the discordance between direct and indirect comparisons is small.

The introduction of a duplex potentiometer, reducing one tone as the other is increased, enables the observations to be made more rapidly and with greater confidence. After each balance, the relays are stopped and the necessary current readings taken. In addition, the observer is given an opportunity to compare the loudnesses of the tones he has adjusted and is asked if his ear accepts them as equal. In nearly every case he finds no fault with the balance. The oscillators are then interchanged on the potentiometer and the observation repeated; the average of the two settings is plotted. A discordance as great as 5 per cent. is uncommon, and does not occur in the absence of distracting room noises.

Suitable changes in currents enable four pairs of balances to be made at levels such that the corresponding points are uniformly spaced on a logarithmic plot.

EAR SURVEYS WITH THERMOPHONE.

A survey of 20 ears (of 10 men and 10 women) was carried out with the thermophone. Comparisons were made at four levels involving the frequencies 100 (or 120), 200, 300, 500, 700, 1,000, 1,500, 2,000, 3,000 and 4,000 cycles. Time did not permit all possible combinations, hence in several cases comparisons were made of widely separated frequencies in order to test the claim that direct comparisons may be inferred from indirect.

Of 180 comparisons made by the 20 observers, the results in 93 cases fitted logarithmic straight lines to within the error of the current measurements. No systematic deviations, either for men or women, were found in the other cases. Accordingly, the occasional wide deviations were attributed to experimental blunders. In view of the general confidence in Fechner's law and the concordance with it of most of the results of this survey, the logarithmic equation seems satisfactory.

As an illustration, the results obtained by one observer are given in the table below:

TABLE II.

Observer: F. H. G.

$\log_{10} P_1 = A + B \log_{10} P_2$.

	f_1.	f_2.	A.	B.
(1).....	100	300	0.63	.92
(2).....	200	300	.19	1.00
(3).....	300	500	.30	1.04
(4).....	500	700	.21	.97
(5).....	700	1000	.14	.97
(6).....	1000	1500	.30	1.02
(7).....	1500	2000	.08	.96
(8).....	2000	3000	.13	.93
(9).....	3000	4000	.28	.90
	200	3000	1.41	.91 direct
	200	3000	1.34	.91 indirect combining (2)-(3)-(4)-(5)-(6)-(7)-(8)

Weighting the observations equally, we plot in Fig. 5 the averages of all. For each comparison four circles, each the average of 20 settings, represent the observations by the men; four crosses, those by the women.

It is apparent that the averages fit logarithmic straight lines closely and that there is no certain difference between circles and crosses, except in the comparisons of 2,000 and 4,000 against 3,000. It is near these

frequencies that resonance is to be expected. The canal and the ear cavity have been taken in the pressure computations as of the *average* length and volume; for this reason the circles and the crosses would fail to coincide at frequencies near resonance should there be between men and women a systematic difference in shape of ear cavity.

Table III. is a condensed tabulation of the results obtained. The separate comparisons have been reduced to 700 cycles as reference frequency.

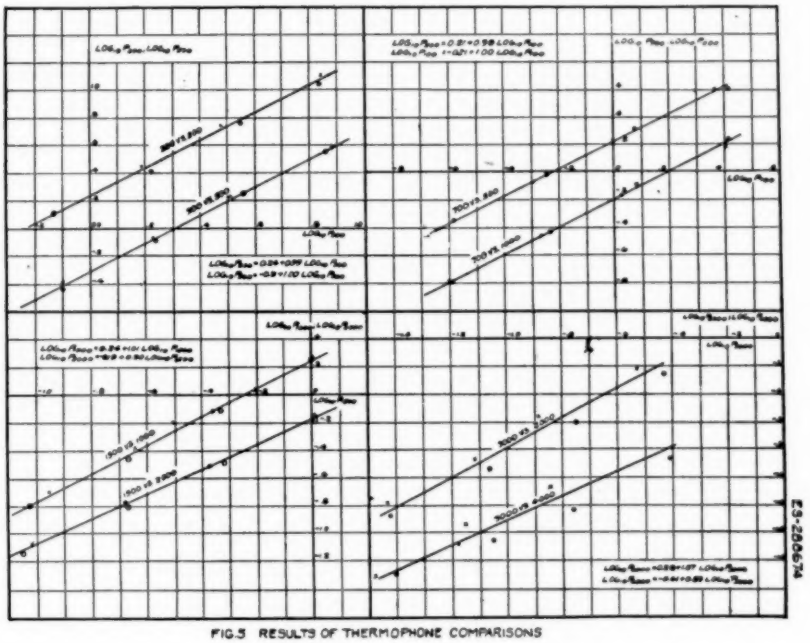

FIG.5 RESULTS OF THERMOPHONE COMPARISONS

Fig. 5.

TABLE III.

$\log_{10} P_f = A + B \log_{10} P_{700}$.

f.	*A*.	*B*.	Individual Extremes.	
			A.	*B*.
200.....	0.76	0.97	0.57, .96	0.62, 1.42
300.....	0.52	.98	.41, .65	.59, 1.18
500.....	0.21	.98	.14, .33	.77, 1.10
700.....	0.00	1.00	— —	— —
1,000.....	−0.21	1.00	− .29, −.09	.83, 1.29
1,500.....	−0.45	.99	− .66, −.33	.80, 1.22
2,000.....	−0.60	.89	− .81, −.38	.62, 1.09
3,000.....	−0.82	.84	−1.40, −.50	.63, 1.14
4,000.....	−1.14	.74	−1.67, −.79	.39, 1.37

RELATIVE SENSITIVITY OF THE EAR.

It is convenient to regard the data, taken all together, as yielding a determination of the acoustic pressures which phonometrically balance the pressure of 1 dyne per sq. cm. at 700 cycles. The reciprocals of these pressures express the relative ear sensitivity. The frequency of 700 cycles is chosen as standard because it is the last audible frequency in the average voice progressively attenuated, and the reference value of pressure roughly corresponds to the band of average speech between 600 and 800 cycles.

In Fig. 6, curve A is the ear *sensitivity* referred to the standard just described. The ordinates of this curve are the negatives of the values of A in Table III.; frequencies on the musical scale are plotted as abscissæ.

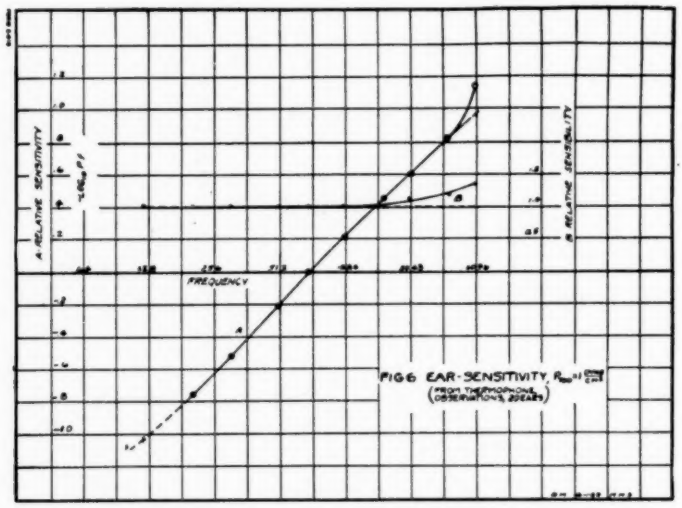

Fig. 6.

Values of B in Table III. show the change in log P_f required to maintain the phonometric balance when log P_{700} changes by unity. Reciprocals of the B's therefore exhibit the ear *sensibility* referred to 700 cycles: curve B, Fig. 6.

It is to be emphasized that the alternating pressures generated by the thermophone are obtained from computation. As yet no method has been devised for the immediate measurement of pressures on the ear drum, or for studying the reaction of the air chamber and walls of the ear on the thermophone. Until such ideal measurement is possible, the values of A cannot be guaranteed free from systematic errors. Prob-

ably these errors are small, and below 2,000 cycles do not greatly vary with frequency. The abrupt turn in curve A, Fig. 6, may be due either to resonance of the air cavity or to resonance of the drum.

The value of B, on the other hand, should be unaffected by an uncertainty in that of A, provided only the acoustic output of the sound generator be proportional to the electrical input. It will be noted that up to 1,500 cycles, the relative *sensibility* is unity, but increases appreciably to 1.35 at 4,000 cycles. Reëxamination of the data discloses the fact that if we exclude those observers whose comparisons above 1,000 cycles were in any case erratic, we have left eight ears out of twenty for which the values of B lie between 0.9 and 1.1. The indirect comparison of 4,000 and 700 involves five steps: blunders or errors in any one of these distort the series of B's for the ear concerned. It appears reasonable, therefore, to take the sensibility as uniform over the frequency range covered.[1]

In Fig. 6 observations at 100 cycles by 8 men and at 120 cycles by the 10 women are represented by crosses, to which the curve A is tentatively extended.

The advantages of the thermophone as a sound generator are convenience of computation and practical absence of transients. A serious disadvantage is the narrowness of the available range of loudness. The extremes of pressure within which the ear can accommodate are in the ratio 10^6 to 1; the thermophone surveys cover only one-sixth of this range, the pressure limits being 10 to 1. Conceivably, the simple law valid within these limits may fail outside them.

Extension of Loudness Range.

Wider limits were reached by the use of an air-damped electromagnetic receiver in place of the thermophone. The supply circuit of the receiver includes an attenuator, as shown in Fig. 2. The dial of the attenuator is marked in "napiers": the introduction of N napiers reduces the voltage at the receiver terminals to e^{-N} times its former value. On this account it is convenient to express the sound output of the receiver in napiers (natural logarithms of the pressures).

The unattenuated sound emitted by the receiver is as loud as the ear can tolerate without discomfort. With the attenuator the tone can be reduced to inaudibility.

The electromagnetic receiver is calibrated in terms of acoustic pressure per volt input at the frequencies selected for the phonometer comparisons.

[1] This conclusion is confirmed by V. O. Knudsen, who finds the sensibility nearly independent of the frequency between 100 and 4,000 cycles: "Sensibility of the Ear to Small Differences in Intensity and Frequency" (abstract), PHYS. REV., 19, p. 261, Mar., 1922.

An "artificial ear," coupling the receiver and an electrostatic transmitter-amplifier system, enables the calibration to be made as follows:

An alternating voltage of a given frequency and appropriate magnitude is impressed on the receiver terminals and the corresponding amplifier output noted. The receiver is then silenced and a known voltage of the same frequency introduced into the amplifier input in series with the electrostatic transmitter. Comparison of the amplifier output in this case with that due to sound pressure on the transmitter diaphragm, together with the dynamical characteristic of the transmitter, permits computation of the sound pressure corresponding to the alternating voltage on the receiver. The artificial ear is intended to simulate the real ear in respect to volume and mechanical reaction on the receiver diaphragm. Were this simulation perfect, the observations just described would enable us to compute the pressures on the ear-drum when the receiver is held against the ear.

However, the simulation is far from perfect. The real ear affects the motion of the receiver in a way which the artificial ear imitates fairly well over the central region of the frequency range. But the cylindrical cavity and the rigid walls of the artificial ear differ greatly from the comparatively soft walls of the curved auditory canal and external ear, and the artificial ear fails at high and at low frequencies.

We have, then, a calibration of the electromagnetic receiver under conditions which differ from those of use, and the results of work with such a sound generator can not be stated in ideally accurate numerical values.

It is, however, permissible to use the receiver without the perfect calibration. Provided the pressure output of the receiver is proportional to the voltage at its terminals, and provided further that this voltage is known, it is possible to study variations in relative sensitivity as the loudness varies between wide limits.

Both requirements mentioned above may be incompletely satisfied. At high levels the receiver may be overloaded and the sound output cease to be proportional to the voltage. At low levels, where great attenuation is introduced, the impedance of parasite current paths may no longer be large in comparison with that of the network in the attenuator and the voltage at the receiver terminals may be incorrectly deduced from the dial setting. Such parasite paths are offered by capacity between input and output, due to conductors in the vicinity of the attenuator, and by induction owing to inadequate shielding of input and output leads. Care was taken to place and shield the leads safely, but it was desirable to determine how successfully this was done.

The proportionality of sound output to impressed voltage is tested by bringing the receiver up to the transmitter-amplifier and reading amplifier output currents for alternating voltages corresponding to attenuator settings from 0 to 4 napiers. As in the calibration of the receiver, voltages in the transmitter are determined. By reason of the great stiffness and small motion of the transmitter diaphragm, it is proper to assume proportionality of acoustic pressure and voltage generated in the transmitter. It is clear that the load characteristic of the receiver can be derived.

It was found that a correction on this account was not required above 200 cycles. The amplitude of the receiver diaphragm is greatest, and so overloading most easy, at the low frequencies. In the curves later shown the correction at 200 has been applied.

Parasite current paths, the existence of which leads to incorrect estimates of receiver voltage, are important only at low levels. Let the current input to the attenuator be the maximum permissible, and let N_1 be the dial setting necessary to reduce the sound output to the threshold. Now let the input current be reduced in the ratio $1 : e$; N_2 is the new threshold setting. When the parasite paths are negligible, $N_1 - N_2 = 1.0$. The threshold for any individual ear varies from day to day, but is sensibly constant over the few minutes required for this test.

Such an examination failed (for any of the frequencies here considered) to indicate errors exceeding a tenth of a napier,—an admissible experimental error.

Phonometric Comparisons with Electromagnetic Receiver.

Following these studies of the receiver, phonometric comparisons were undertaken by four observers. These comparisons with the electromagnetic receiver extended from 1 or 2 to 10 napiers above the threshold, taking 700 cycles as the reference tone. Each of the other tones was compared directly with 700, avoiding the objectionable features of indirect comparisons.

Before presenting the results obtained, it is profitable to consider what types of curves are to be expected in ideal phonometric comparisons covering a wide range of loudness.

Fig. 7, *I*, shows hypothetical Fechner curves, relating stimulus and "sensation," for the frequencies f_1 and f_2. Such curves for the ear have not yet been determined by experiment, but in drawing the hypothetical figures analogy between eye and ear has been presupposed.

Fechner curves for vision are constructed by integrating curves of sensibility (reciprocal of least perceptible stimulus increment) *vs.* log-

arithm of stimulus. Each curve contains (1) a curved portion at low intensities, concave toward the axis of sensation; (2) a straight portion, throughout which the least perceptible stimulus increment is constant; (3) another curved portion at high levels, concave toward the axis of log stimulus. At each end of the straight portion the curve bends because the least perceptible stimulus increment is no longer constant. Implicit in the construction of these curves is the assumption that whatever be the least perceptible change in the stimulus the accompanying change in the sensation is of invariable amount.

The sensation scale is undetermined, but we can imagine that by an ideal loudness balance, say at the level "a," coincidence of the scales for f_1 and f_2 has been established.

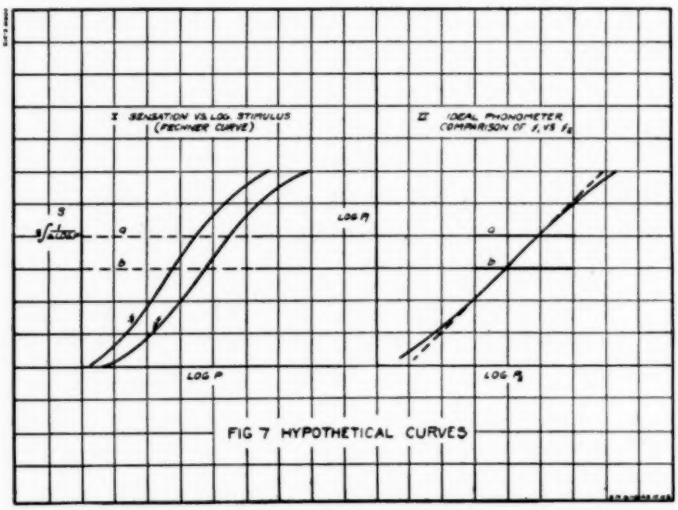

Fig. 7.

If phonometric comparisons are made of the tones represented by these curves, we shall obtain pairs of values of log P_1 and log P_2,—each pair will correspond to a definite sensation level, say "a" or "b." Fig. 7, *II*, shows, in the full line, the ideal phonometer results. In the figure to the left, the curves for f_1 and f_2 start from different thresholds and have different curvatures in the upper and lower portions. Centrally, where each curve is straight, phonometric comparisons will give a straight line for log P_1 *vs.* log P_2. If the slopes of the straight-line portions are the same, the slope of the phonometric line will be unity within these limits. At levels of loudness above "a" or below "b" however, the phonometric line will be curved as drawn. Were the sensation-stimulus curves at all ordinates uniformly spaced in abscissae, the phonometer

would give the dotted line at 45°, coincident in the middle of the range with the full line.

Individual ears may be expected to vary in several respects. The threshold values of stimulus intensity vary with frequency for a given observer, and at each frequency differ with different ears. The values of stimulus intensity at which the Fechner curves become straight lines, and the stimulus intervals within which the lines remain straight, vary among frequencies for each observer and among observers for each frequency. Moreover, the characteristics of the ear change from day to day.

From the foregoing discussion, it is clear that we may expect two types of phonometric curves, provided the analogy holds between the eye and the ear. The types are (1) straight lines over a considerable range; (2) sharply curved lines of which the upper and lower halves are of opposite curvature. Lines of intermediate shapes will of course occur.

The observers who took part in the studies with the electromagnetic receiver are too few in number to make us sure that individual pecularities will be masked in the final average. Besides, the observing program covered six or eight hours for each ear (as against two or three hours in the thermophone surveys) and in three cases the observer was prevented from completing the program the same day he began it. It is therefore desirable to discuss the comparisons separately.

On examination, it was found that the observer who concluded the program in a single day obtained, for all comparisons, sensibly straight lines over the entire range. The three other observers obtained straight lines in most comparisons, but also a number of sharply curved lines of the second type anticipated above.

The distribution of these types and the general aspect of the data may be seen from Table IV. in which are tabulated the frequencies (each compared directly with 700 cycles) and the corresponding numbers of lines of each type.

TABLE IV.

f.	Sensibly Straight.	Sharply Curved.
200	4	0
300	3	1
500	4	0
1,000	2	2
1,500	2	2
2,000	4	0
3,000	3	1
4,000	3	1
	25	7

The preponderance of straight lines is unmistakable. In Fig. 8, at the left, the sensibly straight lines for each frequency are consolidated into an average line; at the right are drawn the individual curves which conspicuously depart from straightness.

Threshold settings were made for each frequency. These are plotted in Fig. 8 and the phonometer curves extended to them. The curves suggest that the auditory accommodation taking place in the comparisons is the same as at the threshold.[1]

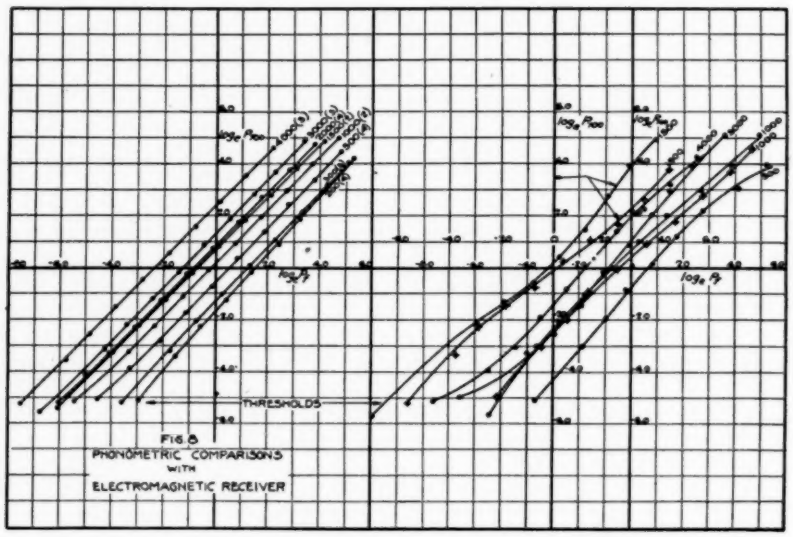

Fig. 8.

A short study was made by one observer, making on different days repeated comparisons of the same two tones. As might be expected, it was found that the curve obtained varied from straight on one day to curved on the next.

Care was taken by two observers to make a partial check of the validity of the phonometric technique by comparing two sounds of the same frequency (700 cycles). Whatever the calibration of the apparatus, equality of loudness in this case should mean equality of currents through the two contacts of the commutating relay. The greatest deviation from equality was 10 per cent. This occurred at the highest level; at intermediate and low levels the deviations were too small to be detected.

[1] The threshold sensitivity derived from these settings differs only at 3,000 and 4,000 cycles from that given by Fletcher and Wegel ("The Frequency-Sensitivity of Normal Ears," Proc. Nat. Acad. Sci., 8, 1, pp. 5–6, January, 1922). The discrepancy is no greater than the uncertainty of the receiver calibration, ascribed to resonance in the ear near these frequencies.

RELATIVE EAR SENSITIVITY AT DIFFERENT LEVELS OF LOUDNESS.

Without regard to shapes of individual curves, we plot the average curve of each comparison and scale off at various levels of the 700-cycle tone the values of log P for the other frequencies. Doing this at three levels ($\log_e P_{700} = -4.0$, 0.0, and $+4.0$) we plot the values of $\log_e P_f$ with reversed signs and so draw at each level a curve of relative sensitivity of the ear. These are exhibited in Fig. 9, where, as in Fig. 6, the abscissae are frequencies on the musical scale; the ordinates, $-\log_e P$, are indicative of the relative sensitivity.

Obviously, the defects of the receiver calibration affect all these curves in the same way. The straightness of most of the curves shown in Fig. 8, and the closeness of their slopes to unity, insure the parallelism of the curves of Fig. 9.

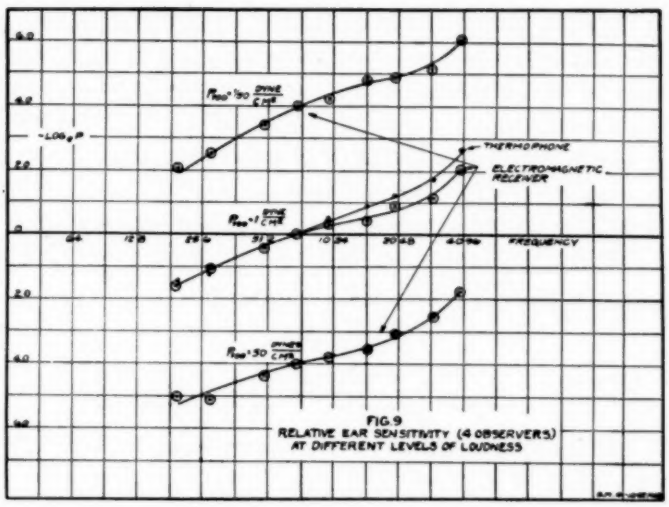

Fig. 9.

The calibration of the receiver and the computation of the thermophone output are both affected by unknown systematic errors. Numerous attempts were made to devise a method of comparing on the ear the sound output of the thermophone with that of the electromagnetic receiver. None of these attempts succeeded. Either the two sound generators must be interchanged on the ear or another receiver must be actuated by them alternately through the transmitter-amplifier system. In the first case it is impossible to make satisfactory comparisons, while in the second case the coupling to the transmitter sustains the uncertainty which the comparison is intended to remove.

As previously explained, the slopes of the lines should be unaffected by errors in the adopted sound pressure-frequency characteristic of either the thermophone or the electromagnetic receiver. From this point of view the two instruments agree: for the four observers using both, slopes by the thermophone over the central portion of the loudness range are 0.9 to 1.0; by the electromagnetic receiver, 0.9 at 200 *vs.* 700, 1.0 elsewhere.

SUMMARY.

1. At an intermediate level of loudness determinations with the thermophone have been made, by 20 observers, of the pressures of certain selected frequencies when these seem to the ear equally loud. The agreement among different ears is so close that improvement in the curve of relative sensitivity shown in Fig. 6 is to be looked for rather from verification of the pressure computations than from extending the survey to include a larger number of observers.

2. A limited number of observers have made observations over a range of loudness from 1 to 10 napiers above the threshold, using an electromagnetic receiver in place of the thermophone as source of sound. Their results are in general agreement with those obtained in the thermophone work, and strongly suggest that the relative ear sensitivity is invariable over this range.

3. The phonometric comparisons have been shown to be in harmony with Fechner's law, the application of this law to the ear being presumed valid.

4. The optical phenomenon known as the Purkinje effect has no analogue within the range of the phonometric comparisons. It is of great interest to continue this work at low intensities. In the present study, room noises made it impossible to work at levels near the threshold, where the Purkinje effect is perhaps to be found.

In conclusion, the results believed to have been accomplished may be thus stated:

I. A phonometer has been devised which permits accurate comparisons of the loudness of pure tones of different pitch.

II. With this instrument, evidence has been found that the relative sensitivity of the ear is independent of the loudness level over a wide range.

RESEARCH LABORATORIES OF THE AMERICAN TELEPHONE AND TELEGRAPH COMPANY,
AND THE WESTERN ELECTRIC COMPANY, INCORPORATED,
April 8, 1922.

THE OPTICAL PROPERTIES OF MOLTEN METALS.

BY A. K. ASTER.

SYNOPSIS.

Optical Properties of Molten Bi, Pb, Sn, and Wood's Alloy to 500° and of Hg to 345° C.—While there is considerable evidence that the optical properties of metals in the solid state do not vary with temperature, the question has not yet been investigated for molten metals. A vacuum tight furnace enabled a nitrogen atmosphere to be used so that surfaces free from oxidation could be studied. The optical properties were determined by letting plane polarized light of wave-length about 6020 Å.U. fall at an angle of 45° upon the metal surface, and analyzing the reflected light with the help of a Jellet split nichol having a Brace elliptic half-shade mounted just before it. The method of calibration and use of the apparatus is described. The optical properties were found to be independent of the temperature for all the metals tried. The mean values, correct to better than one per cent., are as follows: for Bi, $\nu = 0.493$, $k = 1.32$; for Hg, $\nu = 0.442$, $k = 1.43$; for Pb, $\nu = 0.415$, $k = 1.76$; for Sn, $\nu = 0.398$, $k = 1.46$; for Wood's alloy, $\nu = 0.397$, $k = 1.78$. From a discussion it appears that the constancy of the optical properties is not consistent with Drude's equations.

INTRODUCTION AND PREVIOUS WORK.

THE optical properties of metals as a function of the temperature have been investigated to a certain extent for metals in the solid state. Kundt[1] investigated silver, antimony, copper, platinum, iron and nickel between 20 and 100 degrees C., using a direct method, employing thin metallic prisms. He found a slight variation of the index of refraction with the temperature. Drude[2] investigated gold and platinum up to 200 degrees C., and found only a very slight change in the index of refraction, but did not consider the work conclusive. Sissingh[3] investigated iron but did not find any change with the temperature. Zeeman[4] investigated platinum up to 800 degrees C., but did not find any change in the coefficient of reflection. Pfleuger[5] investigated nickel, gold and iron between 20 and 100 degrees C., by the same method as Kundt, direct observations on thin metallic prisms, and did not find any change with the temperature. Very little work has been done on metals in the molten state. C. V. Kent[6] investigated bismuth, cadmium, tin, and lead and their binary alloys. He did not definitely determine whether the optical properties were constant with the temperature or not and

[1] Kundt, Wied. Ann., 36, 834, 1889.
[2] Drude, Wied. Ann., 39, 538, 1890.
[3] Sissingh, Arch. Nierlandaisses, tome XX.
[4] Zeeman, Comm. of the Phys. Lab. Leyden, 20, 1895.
[5] Pfleuger, Wied. Ann., 58, 493, 1896.
[6] Kent, PHYS. REV., XIV., 6, 495.

states in his paper that some work should be done to determine this point. His calculations are on the basis that the optical properties are constant.

It was the object of this investigation to determine whether or not the optical properties of molten metals are functions of the temperature.

DESCRIPTION AND USE OF APPARATUS.

Various methods have been used for determining the constants of elliptically polarized light. The Babinet-compensator and the Babinet-Soleil modification of it have been used for large ellipticities while a more recent development, the Brace elliptic half-shade, has been used for small ellipticities. The apparatus used in this work is of the latter type. The arrangement used was similar to that used by A. Q. Tool [1] in his work on the optical properties of metals. The entire polarizing and analyzing systems were mounted in the arms of a vertical spectrometer especially constructed for the purpose. The polarizing system consisted of the usual collimator with a nicol prism mounted directly after the collimator lens. The analyzing system consisted of a compensator (mica between optical flats) mounted on a circle movable with respect to a fixed vernier and the analyzing nicol, a Jellet split nicol with an elliptic half-shade mounted just before it, was mounted on a second circle movable with respect to a vernier which was fixed with respect to the first circle. The half-shade was set with its dividing line at right angles to that of the split nicol. The entire analyzing system was mounted in the telescope arm of the spectrometer which was fitted with a short focus ocular. The spectrometer was fitted with a special device connecting the telescope and collimator arms so that when the collimator was set for a given angle of incidence the telescope was automatically set in the proper position for the reflected beam.

The source of light was a spectroscope fitted with a constant deviation prism and illuminated with an arc lamp using specially treated commercial carbons giving a strong band at approximately 6020 Å.U.

The theory of this analyzing system has been given by Tuckerman[2] in an article on elliptic analyzing systems. The procedure is to obtain a match of all four parts of the field and then make the complementary setting. Let R and r be the positions of the compensator and the nicol respectively for a match, and R' and r' for the complementary match and

$$R' - R = c,$$
$$r' - r = n;$$

[1] Tool, PHYS. REV., XXXI., 1, 1.

[2] Tuckerman, Univ. of Neb. Studies, IX., 2, 157.

then if plane polarized light is let fall on the analyzing system, Tuckerman finds that

$$\cos 2\Pi N_1 = \frac{\tan c}{\tan n}, \tag{1}$$

$$\tan 2\eta = \sqrt{\frac{\sin (n - c) \sin (n + c)}{\sin c}}, \tag{2}$$

where 2η is the effective order of the half-shade and $2\Pi N_1$ is the order of the compensator. If the initial light is not plane polarized, he finds that

$$\tan 2\Pi N_1 = \sqrt{\frac{\sin (c - n)}{\sin (c + n)}} (1 - \alpha \tan 2\eta + 1/2\alpha^2 \tan^2 2\eta - \cdots), \tag{3}$$

$$\tan 2\psi = \sqrt{\frac{\sin (c - n) \sin (c + n)}{\sin n}} (1 - 1/2\alpha^2 \tan^2 2\eta - \cdots) \tag{4}$$

where

$$\alpha = \frac{\sin c}{\sqrt{\sin (c - n) \sin (c + n)}} \tag{5}$$

and if $2\gamma = (R' - R) - (R_0' - R_0) - 2\Phi$ and Φ is 45° as is customarily used, then

$$v_\theta = - \frac{\sin \Phi \tan \Phi \cos 2\psi \cos 2\psi}{1 - \cos 2\psi \sin 2\gamma}, \tag{6}$$

$$K_\theta = - \frac{\tan 2\psi}{\cos^2 \gamma}, \tag{7}$$

where R_0' and R_0 are the settings on plane polarized light,

v_θ = index of refraction,
K_θ = coefficient of absorption,
θ = angle of incidence.

By letting plane polarized light fall on the analyzing system, taking the necessary readings and applying equations 1 and 2, it is possible simultaneously to calibrate the compensator and the half-shade. By analyzing plane polarized light (for $\Phi = 45°$) when it is reflected from the metal surface and applying equations 4, 5, 6 and 7 the optical properties for that angle of incidence and wave length of light used, can be obtained.

EXPERIMENTAL WORK.

The nicol prism of the polarizer was first adjusted for an azimuth of 45° by means of an auxiliary nicol. The compensator and elliptic half-shade were next simultaneously calibrated. This was done by setting the analyzer in direct line with the polarizer and making the two settings

for a match. Substitution of the data thus obtained in equations 1 and 2 gives the following results:

$$\cos 2\Pi N_1 = 0, \qquad 2\Pi N_1 = 90°,$$
$$\tan 2\eta = 0.366, \qquad 2\eta = 20.1°.$$

A special vacuum-tight furnace was designed because in order to prevent oxides from forming on the surface of the molten metal it was necessary to have the furnace filled with nitrogen. A reducing atmosphere would probably have been better, but the risk of an explosion with the use of hydrogen was considered too great to warrant using it, so an inert gas was resorted to. Ordinary commercial nitrogen was found satisfactory. The commercial product contains about 5 per cent. of oxygen according to the manufacturer. In order to keep the amount of oxygen a minimum, the nitrogen was kept in the furnace at a pressure just high enough to prevent boiling of the metal. This pressure was from 20 to 30 cms. of mercury. The remaining oxygen was easily removed by skimming the metal a few times after it was molten, thus converting the oxygen to metallic oxide. After a few skimmings, it was possible to keep the surface clean for a considerable time. It should be stated at this point that the furnace was first exhausted before filling with nitrogen. It was also washed several times before the final filling with nitrogen, this being done to reduce the amount of oxygen to a minimum. This entire operation was completed before the furnace was heated to a point where the metal melted. It was necessary to have the furnace at least 25 degrees above the room temperature while exhausting it, in order to drive out most of the residual gas in the magnesia packing, etc.

The windows of the furnace were constructed of optical glass which was tested so as to be sure that it was free from optical strain. It was necessary to limit the observations to one angle of incidence (45°) in order that the entering and emerging light should always be normal to the plane of the windows to prevent introduction of errors. To be able to vary the angle of incidence it would have been necessary to reconstruct the major portion of the furnace housing for each angle. The furnace was mounted on a special anti-vibration mounting in order to prevent the vibration of the building from disturbing the surface of the molten metal while observations were being taken. This special mounting consisted of a box containing sand, on top of which were placed several marble plates separated by rubber stoppers. The furnace was set on top of the marble plates. The temperature of the metal was measured by means of a chromel-alumel thermocouple. The cold juncture was

kept in a Dewar flask filled with distilled water, the temperature of which was measured with a good mercurial thermometer. A Leeds-Northup potentiometer type pyrometer indicator reading directly in millivolts was used as the indicator. The calibration of the couple was taken from data available in the laboratory. See Fig. 1 for a cross-section of the furnace.

A number of metals were investigated, the results of which are listed below. It was found impossible to continue the investigation above about 500 degrees C., as at this point the metal begins to glow a dull cherry red and this light masks the light from the collimator so that on further settings could be made. It was impossible to use filters as the two colors were nearly alike and any filter which would exclude the one beam would so reduce the other that it would be impossible to make accurate settings and in this sort of work it is necessary to have the maximum possible intensity so as to insure sufficient sensitivity of the apparatus. The sensitivity of the system drops off very rapidly with diminishing intensity. It was found impossible to make any determinations just at the melting point because at this point the metal surface is still rough. It was found necessary to raise the temperature at least about ten degrees above the melting point of the metal to get the metal liquid so as to get a smooth surface. How far one has to go above the melting point depends on the metal. It was found impossible to take any data above 345 degrees C. for mercury, because above this point the mercury vapor condensed on the windows of the furnace making it impossible to see through them. Two attempts were made to get data for cadmium but both were failures. In the first attempt, the cadmium combined with the graphite crucible, and it was impossible to get a clean surface. In the second trial an alundum crucible was substituted for the graphite one, but it was still found impossible to get a clean surface. (Ordinary chemically pure metals were used for all tests.)

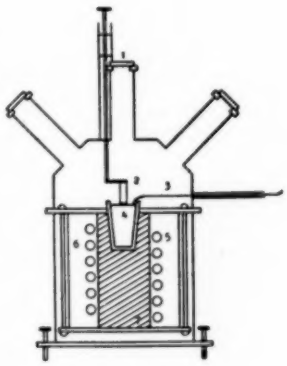

Fig. 1.

1, Glass window. 2, Stirrer. 3, Thermocouple. 4, Crucible and metal. 5, Heater conductor. 6, Magnesia packing. 7, Iron heater core.

RESULTS.

Lead.

Temp.	ν_θ.	K_θ.
335° C.	0.414	1.76
345	0.413	1.74
365	0.415	1.75
385	0.417	1.78
405	0.414	1.77
427	0.416	1.74
440	0.414	1.78
475	0.415	1.77
485	0.416	1.76
Mean	0.415±0.0006	1.76±0.005

Tin.

Temp.	ν_θ	K_θ.
250° C.	0.396	1.46
315	0.398	1.44
335	0.397	1.48
345	0.399	1.45
357	0.400	1.46
375	0.397	1.47
395	0.398	1.44
425	0.396	1.46
450	0.400	1.48
480	0.398	1.44
Mean	0.398±0.0005	1.46±0.0045

Bismuth.

Temp.	ν_θ.	K_θ.
275° C.	0.493	1.31
285	0.492	1.32
295	0.494	1.31
317	0.491	1.30
345	0.495	1.34
375	0.491	1.33
395	0.493	1.32
405	0.492	1.34
425	0.494	1.31
450	0.491	1.33
465	0.493	1.32
485	0.495	1.32
Mean	0.493±0.0005	1.32±0.006

Mercury.

Temp.	ν_θ.	K_θ.
22° C.	0.441	1.42
65	0.442	1.43
125	0.444	1.45
175	0.440	1.41
212	0.441	1.41
265	0.443	1.43
285	0.443	1.42
300	0.440	1.45
325	0.444	1.41
345	0.444	1.43
Mean	0.442±0.0004	1.43±0.0045

Wood's Alloy.

Temp.	ν_θ.	K_θ.
100° C.	0.397	1.80
125	0.400	1.78
150	0.395	1.76
175	0.396	1.77
200	0.398	1.76
220	0.399	1.80
242	0.395	1.78
265	0.397	1.79
285	0.396	1.79
305	0.398	1.78
325	0.395	1.76
345	0.399	1.76
365	0.400	1.80
385	0.397	1.76
407	0.397	1.78
425	0.398	1.77
450	0.395	1.77
465	0.398	1.78
480	0.397	1.76
Mean	0.397±0.00045	1.78±0.0047

DISCUSSION OF RESULTS.

From the results, it is obvious that within the limits of the investigation and the sensitivity of the apparatus used, the optical properties of the metals investigated are constant.

Since the optical properties of Sn, Pb, and Bi are constant and the factors for Wood's Alloy are constant, and Wood's Alloy is made of Sn, Pb, Bi, and Cd, it seems reasonable to conclude that the optical properties of cadmium are constant.

With the view of possibly making a comparison between the factors of the molten and the solid state some observations were made on a highly oxidizable metal (copper). It was found that the age of the surface after polishing as well as the kind of abrasive used in making the mirror greatly affected the results so that no consistent data could be obtained. From this it was concluded that it would be necessary to prepare the mirror in a reducing atmosphere and without the use of an abrasive if accurate results are to be obtained. To prepare a surface under these conditions seems practically impossible. This condition prevented any comparison of the data for the two states. This difficulty applies only to soft, low melting point, highly oxidizable metals and the other metals could not be investigated because their melting points are above 500 degrees C. This point is the upper limit of the work as was previously stated.

PROBABLE ERROR.

The least squares solutions of the experimental results show that the probable error of these quantities is under one half of one per cent. The probable error of the adjustment of the polarizing and analyzing prisms with respect to their circles was well under one-half of one per cent. It seems therefore probable that the final values of these constants are not in error by more than one per cent.

THEORETICAL.

Drude's equations[1] for the optical constants of metals are:

$$\nu^2(1-\kappa^2) = 1 + \sum \frac{\Phi'}{1-\left(\frac{\tau_h}{\tau}\right)^2} - 4\pi \sum \frac{m'N}{r^2+\left(\frac{m'}{\tau}\right)^2} \tag{8}$$

$$2\nu^2\kappa = 4\pi\tau \sum \frac{Nr}{r^2+\left(\frac{m'}{\tau}\right)^2}, \tag{9}$$

where τ_h = natural period of the electrons of class h.

τ = natural period of the incident light.

N = number of electrons per cubic centimeter.

$m' : me$ $\quad$ m = inert mass of the electron.

e = charge on an electron.

$r = \frac{N}{\sigma}$ $\quad$ σ = electrical conductivity.

Φ' = proportional to the displacement of the electrons from their positions due to the influence of a constant force. For conducting electrons Φ' is ∞.

[1] Drude, Theory of Optics, translation by Mann, p. 398.

The experimental work shows that ν_θ and K_θ are constant. Indicating all the constant terms in equations (8) and (9) as such, they become:

$$C = 1 + \sum \frac{\Phi'}{1 - \left(\frac{\tau_h}{\tau}\right)^2} - E \sum \frac{F}{\frac{N}{\sigma^2} + \frac{1}{N} G^2} \tag{10}$$

$$H = J \sum \frac{1}{\frac{1}{\sigma} + \frac{\sigma}{N^2} P} \tag{11}$$

where C, E, F, G, H, J and P are constants. If equation (11) is true then:

$$\left| \frac{1}{\sigma} + \frac{\sigma P}{N^2} \right|_{t_1} = \left| \frac{1}{\sigma} + \frac{\sigma P}{N^2} \right|_{t_2}. \tag{12}$$

Since σ and N vary with the temperature, this identity must hold for all corresponding pairs of values of σ and N. To test this identity, data for mercury will be substituted.

For 0° C. $1/\sigma = 94 \times 10^{-6}$,
350° C. $1/\sigma = 135.5 \times 10^{-6}$,

N varies inversely as the expansion of the metal.

For 350° C. Let $N = 1$.
0° C. $N = 1.064$.

Substituting in equation (12), neglecting the first term on each side of the equation (this is possible because of their size in comparison with the following terms) and inverting, the result is:

$$106 = 135,$$

which is obviously not true, the difference being about 21.5 per cent. Since this difference is far greater than the experimental error of the work, the validity of equation 11 seems doubtful. As to equation 10 very little can be said as no data is available as to the variation of Φ' and τ_h with the temperature.

In conclusion, I wish to thank Professors Hall and Minor and all others whose kind assistance made this work possible.

PHYSICAL LABORATORY,
UNIVERSITY OF CALIFORNIA,
April, 1922.

BOMBARDMENT OF METAL SURFACES BY SLOW-MOVING ELECTRONS.

By H. E. Farnsworth.

Synopsis.

Secondary Electrons Produced by Electronic Bombardment of Nickel.—The main features of the apparatus used are: an equi-potential nitrate-coated Pt cathode heated by radiation from a tungsten spiral filament; a series of insulated diaphragms to limit the beam of primary rays and for use in determining their velocity; and a long Faraday cylinder in front of which the nickel target could be alternately interposed and withdrawn. By baking out the tube and using liquid air traps an extremely low vacuum corresponding to less than 10^{-7} mm. was attained. The secondary current was determined from the difference between the current to the Faraday cylinder and that to the interposed target. (1) *Ratio of the secondary (emergent) to the primary (incident) electron current* was found to be independent of the roughness of the surface, but to vary with the treatment. After heating the target red-hot by high frequency induction for some minutes, a limiting curve was reached which probably represents the characteristics of *nickel itself, free from surface contamination.* In this case the secondary electrons begin when the primary velocity is as low as 0.2 volt; the ratio to primary current then increases rapidly with primary velocity to about 4 volts, remains constant to about 9 volts, then again increases reaching a value of unity for a primary velocity corresponding to about 260 volts. *The effect of exposure to air or hydrogen* is to increase very considerably the secondary current and to round out the flat part of the curve between 4 and 9 volts. (2) *Velocity distribution curves of the secondary electrons* indicate that for primary electrons of less than 9 volts velocity, most of the secondary electrons have velocities nearly equal to the primary velocity, while for primary velocities above 9 volts, the percentage of secondary electrons having small velocities increases with the primary velocity, although a small proportion have velocities nearly equal to the primary velocity up to at least 110 volts.

Reflection and Emission of Electrons from a Nickel Surface Bombarded with Electrons of Velocity 0 to 260 Volts.—It is suggested by the above results, that reflection occurs for all the primary velocities investigated, and that emission or ionization begins at about 9 volts and increases with the primary velocity.

Contact difference of potential between a gas-free nickel surface and an ordinary baked nickel surface was found to be 0.8 volt.

Method of making a gas-tight glass joint for high vacuum work by soldering with Wood's metal, the platinized and copper-coated surfaces of a ground joint, is described.

Introduction.

STUDIES of the secondary electrons produced by electronic bombardment of metal surfaces have previously been made by various experimenters. However, the fact that most of the results were obtained previous to the development of modern high-vacuum technique combined with the failure of the small amount of recent work to agree with these older results, leaves the important questions of this problem still

unanswered. It is well known that the number of secondary electrons depends upon the velocity of the incident or primary beam, that more electrons leave the surface than strike it if the primary velocity is great enough, but the following characteristics of various metals are still not definitely determined: (1) the magnitude of the secondary electron current as a function of the primary velocity; (2) the velocity distribution of the secondary electrons for any given primary velocity.

Some of the earlier experiments carried out by Gehrts,[1] von Baeyer,[2] and Campbell [3] indicate that the above characteristics are the same for all metals tried, platinum, copper, nickel, aluminum, lead and cobalt. Secondary electrons were found when using a primary velocity as low as one volt. The number of secondary electrons increased with increase of the primary velocity up to about 5 volts. With further increase of primary velocity to about 11 volts, the secondary current decreased. Beyond 11 volts it increased rapidly to a maximum at about 200 volts and then decreased gradually and continuously. Above about 30 volts primary velocity more electrons were found to leave the surface than strike it. Most of the secondary electrons apparently had velocities less than 10 volts while none were found with velocities greater than 30 volts even when the primary velocity considerably exceeded 30 volts.

A. W. Hull,[4] working more recently with copper, obtained a slight minimum in his secondary electron curve, but it comes at 5 volts instead of at 11 volts, with a slight maximum at 2.6 volts. He found the ratio of the secondary electron current to the primary, for any given primary velocity, to be much smaller than that of the previous experimenters, being only about 40 per cent. at 50 volts primary velocity. I. G. Barber,[5] also working recently with copper, concluded that there were no secondary electrons when the primary velocity was less than about 10 volts. F. Horton and Miss A. C. Davies[6] found secondary electrons from platinum when the primary velocity was as low as 3 volts. They found no secondary electrons with velocities greater than 10 volts when using a primary velocity of 70 volts. They did not heat the platinum above 300° C. Davisson and Kunsman,[7] working with nickel, found some secondary electrons with velocities nearly equal to that of the primary beam. They

[1] Gehrts, Ann. d. Phys., *36*, p. 995, 1911.

[2] O. von Baeyer, Phys. Zeitschr., *10*, p. 176, 1909; Deutsch. Phys. Gesell., Verh., *10*, 24, p. 953, 1908; Ber. d. D. Phys. Ges., *10*, p. 96 and p. 953, 1910.

[3] N. R. Campbell, Phil. Mag., *22*, p. 276, 1911; *24*, p. 527, 1912; *25*, p. 803, 1913; *28*, p. 286, 1914; *29*, p. 369, 1915.

[4] A. W. Hull, PHYS. REV., *7*, p. 1, 1916.

[5] I. G. Barber, PHYS. REV., *17*, p. 322, 1921.

[6] F. Horton and Miss A. C. Davies, Royal Soc. Proc., *97A*, p. 23, 1920.

[7] Davisson and Kunsman, Science, *54*, p. 522, 1921.

did not investigate the number of secondary electrons as a function of the primary velocity.

The object of the present investigation is to obtain the characteristics of various metals which have been freed from gas under the best vacuum conditions available, and to see how these characteristics vary with the metal and the roughness of the surface. Up to the present only nickel has been studied in detail.

APPARATUS.

An apparatus suitable for the carrying out of this investigation must be so constructed that the following conditions may be realized: (1) a primary electron stream of uniform velocity and cross-section; (2) an accurate means of measuring the total primary current; (3) an accurate means of measuring the secondary electron current; (4) means of measuring velocities of both primary and secondary electrons.

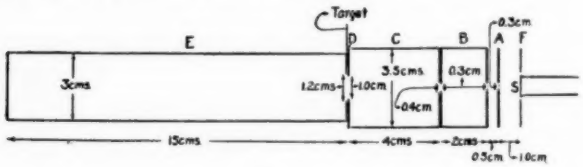

Fig. 1.

The final form of apparatus used is shown in cross-section in Fig. 1. Condition (1) is satisfied by accelerating the primary electrons to the diaphragm, A, from an equipotential source, S. Some of these electrons shoot through the hole in A, through B and C into E. The primary beam is thus limited in cross-section by the various sharp-edged diaphragms along the path. F is a focusing plate concentric with S. The electron stream is kept from scattering by means of a magnetic field in the direction of the stream. Condition (2) is realized by the use of the long cylinder, E, which is practically a perfect absorber for the electrons that once get into it. (3) By means of a magnetic control [1] a nickel target can be moved over the hole in the end of E, into the path of the electron stream. The secondary electrons which leave the target are received by the cylinder C. The insulated diaphragm D makes it possible to determine whether the presence of the target causes any scattering of the primary beam. (4) The velocity distribution of the primary electrons can be obtained by putting a variable retarding potential on D and E and measuring the total current as a function of the potential in the usual way. Similarly, the velocity distribution of the secondary

[1] The magnetic control was placed to the left of the cylinder, E, as far away from the source as possible, to avoid changing the state of magnetization of that part of the apparatus which limits the electron stream.

electrons may be obtained by varying the potential of C and D. The apparatus was constructed entirely of nickel, glass and quartz except for the nitrate-coated platinum source of electrons. A quartz tube, supports the platinum source which is coated with barium, strontium, and calcium nitrates to increase the emission of electrons. The heat for the source is furnished by an electric current which passes through a plane tungsten spiral, placed inside the quartz tube and very close to but not in contact with the platinum foil. With this arrangement the source is always an equipotential one which permits an accurate measurement of its potential. A larger primary current was obtained when fine nickel gauze was placed over the openings in A and B, as indicated in Fig. 1 by the dotted lines. This is probably because there is less distortion of the field at the openings when the gauze is used. The cylinders B, C, and E were lined with fine nickel gauze so as to decrease any possible multiple reflection of electrons. The whole apparatus was cleaned with distilled ether after assembling to remove any grease that might be present. The apparatus was enclosed in a glass tube which, after being evacuated, was heated by an electric furnace to 400° C. for several hours with the pumps working continuously. Liquid air was kept on the condensing trap continually during the time that the tube was connected to the vacuum line, so that oil and mercury vapors were never allowed to enter the tube. The pumps were kept going during the observations so that the results were obtained under extremely good vacuum conditions. A pressure of 10^{-6} mm. Hg could be observed on the McLeod gauge used and while the observations were taken the mercury stuck to the top of the gauge, a negative pressure of 2 cm. of Hg being necessary to release it, so that the gas pressure in the tube must have been much less than 10^{-7} mm. Hg. Observations as well as calculation show that ionization of the residual gas at this pressure is negligible since practically no current to C and D, Fig. 1, was observed when primary electrons were driven into E.

In order to facilitate adjustment and to prevent the waste of time necessitated by cutting open the tube containing the apparatus, a ground glass joint, having the same diameter as the tube (2 inches), was sealed on one end. To make the joint air tight and to avoid the presence of wax vapor, the joint was copper plated and soldered together with Wood's metal. The glass was first platinized by painting on a solution of platinum chloride and dextrine, then heated in an electric furnace until the platinum was reduced to the metallic form. Four or five separate coats and heatings were necessary to make the platinum sufficiently conducting so that a uniform deposit of electrolytic copper could

be obtained. At first difficulty was experienced in obtaining a deposit of copper which did not let air through between the copper and platinum. This was finally overcome by taking special precautions in the cleanliness of the solution of copper sulphate used, and by the addition of a small amount of glucose to the solution, which produced a finer grained deposit. The copper was deposited on the two parts of the joint so that when they were fitted together the copper plating completely covered the outside of the joint. Wood's metal was used to solder the two parts together because of its low melting point. With zinc chloride it required very little heating to coat the surface of the copper with the alloy. The vapor pressure of Wood's metal is not exactly known, but it should cause no trouble in the tube since two inches of tight-fitting ground-glass joint intervene to make the rate of diffusion extremely small. The ground joint was placed near one end of the tube and far enough from the metal parts so that it was possible to bake out the apparatus without heating up the joint. The inner seal which was attached to the removable part of the ground joint supported the wires carrying the heating current and the quartz tube, and also two small glass tubes, one of which held the focusing plate, F, and the other the diaphragm, A.

METHOD OF PROCEDURE.

Since the magnitude of the primary current was found to vary with the velocity of the primary electrons, the ratio of the secondary current to the primary current was measured as a function of the primary velocity. The secondary current was obtained by taking the difference between the total primary current to E when open, and the current to the target when it was over the hole in the end of E. Direct measurement of the secondary electron current to the cylinder, C, gave the same result as the above method. B, C, D, and E were all grounded during this measurement so that the number and energy of the secondary electrons were not affected by an external field. In order to obtain larger primary currents for the small velocities a retarding field was placed between A and B, so that the primary electrons were first accelerated to A, retarded to B, and then passed on into E with a velocity measured by the difference between the accelerating and retarding potentials. The procedure was then to measure the primary and secondary electron current for various primary velocities, obtained by varying the potential of S.

To determine the velocity distribution of the primary electrons, *i.e.*, to find out what fraction of the total number had a velocity corresponding to the net accelerating voltage, a varying retarding potential was put

on E and D, with E open, the potential of the two being the same. This potential was varied in steps from zero up to the accelerating voltage, and the corresponding currents to the cylinder, E, measured. By trial, it was found that no electrons hit D. The velocity distribution of the secondary electrons for any given primary velocity was obtained by measuring the current to the target as the potential of B, C, and D (all being the same) was varied in steps from zero up to the voltage corresponding to the primary velocity. B was kept at the same potential as C and D, for it was found that the primary stream was scattered if there was an appreciable potential gradient between B and C. This procedure is based on the assumption, which seems to be a reasonable one, that the velocity of impact of the primary electrons is given by the potential drop from the target to the source, independent of the potentials of B, C, and D. Changes in the potential of B, C, and D produce changes in the magnitude of the primary electron stream, but these are taken account of, since the total current to E, as well as the current to the target, was measured each time that the potential of B, C, and D was changed. The ratio of the secondary electron current to the total current received by E as a function of the retarding potential on B, C, and D then gave a measure of the velocity distribution of the secondary electrons, since the secondary electrons which leave the target must have a velocity greater than that corresponding to the retarding field which tends to keep them on the target.[1]

RESULTS.

As shown by the curves of Fig. 2, the ratio of the secondary to the primary current as a function of the velocity was obtained for the following: (1) a polished nickel target; (2) a nickel target which had been cleaned with fine emery paper; (3) nickel target etched with acid; (4) nickel target covered with fine nickel gauze; (5) a copper target; (6) a polished nickel target after prolonged heat treatment at bright red heat. The first five curves were obtained with targets which had received the same heat treatment as the rest of the apparatus, *i.e.*, baked at 400° C. for several hours. The results show that the number of secondary electrons is independent of the roughness of the surface, but that covering

[1] The above method assumes, of course, that the retarding potential on the cylinder, C, is a measure of the total velocity, and not of components perpendicular to the surface of the cylinder. This would only be true if a small target was placed at the center of a hollow spherical conductor large compared to the size of the target. However, the error introduced by an apparatus having the dimensions of Fig. 1 should not be serious. In any case, it would be constant and such as to make the value obtained for the velocity smaller than the true value, so the fact that large values were obtained for at least some of the secondary electrons would not be altered.

the surface with fine nickel gauze considerably reduces the number of secondary electrons. The secondary electron current from a copper target seems to be greater than that from a nickel target. However, since the above observations were taken before sufficient heating to eliminate the effects of occluded gas, no final conclusion can be drawn as to the difference in the secondary electron current from copper and nickel.[1]

Prolonged heating of the nickel target at bright red heat was accomplished by high-frequency induction. The target could be moved up into a bulge in the tube so that only the target was heated to red

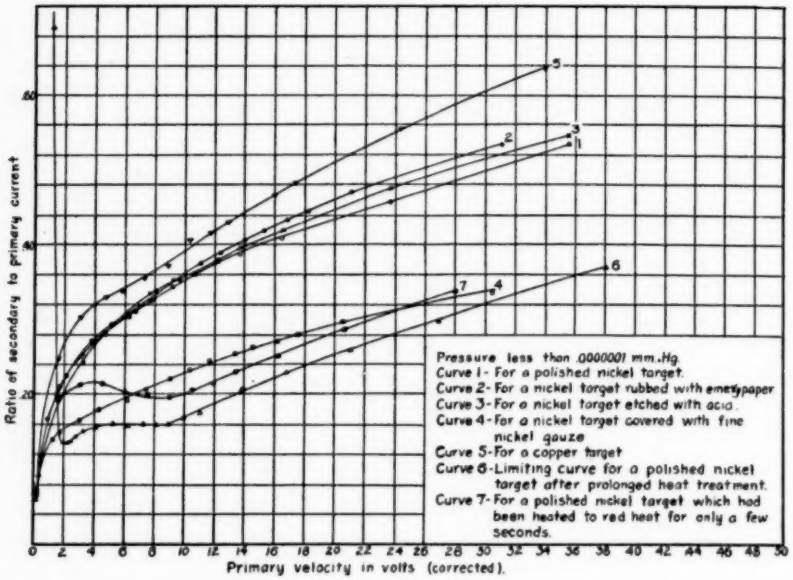
Ratio of secondary to primary current
Primary velocity in volts (corrected).
Pressure less than .0000001 mm. Hg.
Curve 1- For a polished nickel target.
Curve 2- For a nickel target rubbed with emery paper
Curve 3- For a nickel target etched with acid.
Curve 4- For a nickel target covered with fine nickel gauze
Curve 5- For a copper target
Curve 6- Limiting curve for a polished nickel target after prolonged heat treatment.
Curve 7- For a polished nickel target which had been heated to red heat for only a few seconds.

Fig. 2.

heat when the tube was surrounded by a few turns of wire which carried the high-frequency current. The appearance of the target was unchanged by the heat treatment, although a considerable amount of nickel was evaporated onto the walls of the glass tube opposite the target. Since the pumps were kept going during this heat treatment the gas pressure remained small at all times (less than .00001 mm. Hg), and when the heating had been continued until the limiting curve was obtained the gas pressure even during the heating was so low that the mercury stuck to the top of the McLeod gauge.

[1] It is interesting to note that a curve of the same form as curve 1 was obtained with a preliminary form of apparatus, in which the nickel target was moved along the axis of the tube, similar to the arrangement used by A. W. Hull.

As shown by curve 6, the prolonged heat treatment reduces the secondary electron current considerably and changes the general shape of the curve. Curve 7 was obtained after the target had been heated to bright red heat for only a few seconds (about 10). Further heating gradually changed and lowered the curve from the form 7 to the form 6. Curve 6 seems to be the limiting value since additional heating produced no further change. To make sure that the change produced was due to elimination of gas and not to a change in the structure of the metal, the apparatus was exposed to air and then evacuated and baked at 400° C., after which the original curve 1 was obtained. Curve 6 was then repeated after prolonged heating. Exposing the apparatus to hydrogen at a pressure of 2 cm. Hg for several days increased the second-

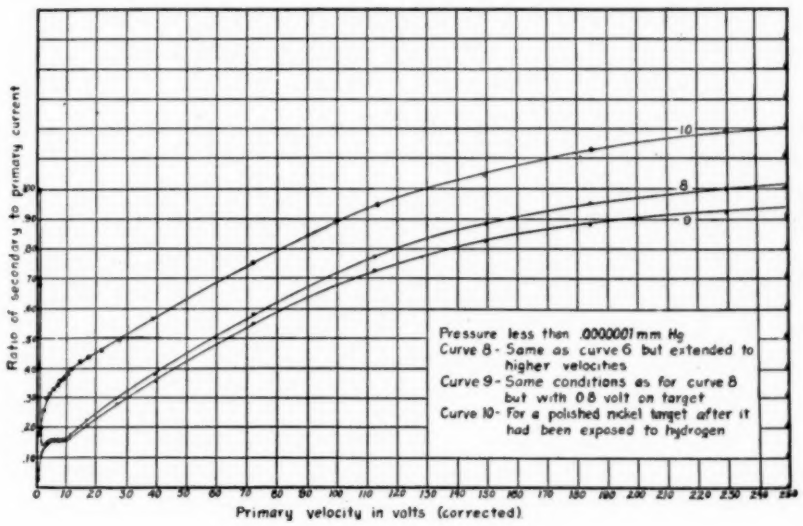
Ratio of secondary to primary current
Primary velocity in volts (corrected)
Pressure less than .0000001 mm Hg
Curve 8 - Same as curve 6 but extended to higher velocities
Curve 9 - Same conditions as for curve 8 but with 0.8 volt on target
Curve 10 - For a polished nickel target after it had been exposed to hydrogen
10
8
9

Fig. 3.

ary electron current as shown by curve 10, Fig. 3, but again heating the target at bright red heat reduced this current to the same limiting values which it reached after heating subsequent to the exposure to air, so that it seems safe to conclude that the final curve is one which represents the true characteristics of the metal.

As already seen from curve 7, an enormous change in the curve was produced by only a few seconds heating of the target at bright red heat, and practically the whole change produced by the continued heating at this temperature was found to take place during the first few minutes. The curve obtained after 30 min. of this heating was only slightly higher than that obtained after one hour of heating. Further heating produced

no noticeable change in the curve, *i.e.*, the limiting form was reached after about one hour of heating at bright red heat. Only about 20 min. of this heating seemed to be necessary to obtain the limiting curve subsequent to the exposure of the apparatus to hydrogen under the conditions previously stated. The heating at red heat was not effected continuously, but at about one-minute intervals for periods of one minute, so as to prevent overheating of the glass tube. No results for a copper target after continued heating have been obtained as yet, due to experimental difficulties.

After the target had been heated to red heat it assumed a negative potential of about 0.8 volt with respect to the surroundings, thus producing the sharp rise at the low-velocity end of the curves obtained after heating. By neutralizing this negative potential with a positive potential of 0.8 volt, curve 9 was obtained. It will be noticed that this curve lies below curve 8, the difference between the two curves increasing with the primary velocity. This, however, is easily explained if we consider that before the contact potential is neutralized the secondary electrons are pulled off the target by a potential difference of 0.8 volt which causes more electrons to leave the target than would if no field existed. This difference would naturally increase with increase in the number of secondary electrons. Curve 9 can then be considered as representing the true secondary electron characteristics of nickel produced in an equipotential region. The curve obtained after allowing the apparatus to stand over night, during which time the pressure remained less than .00001 mm., was found to be somewhat higher than the limiting form, *i.e.*, the secondary electron current had increased slightly. The contact potential of the target had also slightly decreased. This result is probably due to absorption of gas and to gas diffusing out from the interior of the target.

A typical set of data, taken after prolonged heat treatment, is shown in Table I. Column 1 shows the retarding potentials used between *A* and *B*, Fig. 1. These retarding potentials gave the best velocity distribution of the primary electrons, although no difference could be noticed in the curve obtained with a constant retarding potential of 20 volts. Column 2 contains the primary velocity in volts (uncorrected). The total current was measured before and after the current to the target was observed and the average taken as indicated in Column 3. Column 5 contains the current to the diaphragm, *D*, when the cylinder *E* was open and when it was closed by the target. As stated in the description of apparatus, these observations show whether the presence of the target causes any scattering of the primary beam which would result in an

TABLE I.

1	2	3			4	5 (5, a) (5, b)		6	7
Retarding Potential between *A* and *B*.	Velocity in Volts (Uncorrected).	Total Current.			Current to Polished Target.	Current to *D*.		Col. 3 − Col. 4.	Ratio of Secondary to Primary Current.
		First.	Last.	Average.		With Target over Hole in *E*.	With *E* Open.		
3.0	1.0	62.0	62.0	62.0	0.0	0.0	0.0	62.0	1.000
3.0	1.5	162.5	161.5	162.0	1.0	0.0	0.0	161.0	0.994
3.0	1.8	187.5	187.5	187.5	11.5	0.0	0.0	176.0	0.938
3.0	2.0	215.0	222.0	218.5	71.0	0.0	0.0	147.5	0.675
3.0	2.3	234.0	237.0	235.5	193.0	0.0	0.0	42.5	0.181
3.0	2.5	227.0	229.0	228.0	194.0	0.0	0.0	34.0	0.149
3.0	2.7	208.5	208.0	208.2	180.0	0.0	0.0	28.2	0.135
3.0	3.0	218.0	217.0	217.5	188.0	0.0	0.0	29.5	0.136
3.0	3.2	233.0	235.0	234.0	202.0	0.0	0.0	32.0	0.137
3.0	3.5	231.0	233.0	232.0	198.5	0.0	0.0	33.5	0.144
3.0	4.0	232.0	234.0	233.0	197.8	0.5	0.0	35.2	0.151
4.5	5.0	216.0	217.0	216.5	183.8	1.2	0.2	32.7	0.151
6.0	6.0	238.0	238.0	238.0	200.0	1.5	0.3	38.0	0.160
6.0	7.0	225.3	228.0	226.6	191.0	2.0	0.3	35.6	0.157
7.5	8.0	237.0	239.0	238.0	200.0	2.0	0.2	38.0	0.160
7.5	9.0	247.0	247.0	247.0	208.0	2.6	0.2	39.0	0.158
9.0	10.0	235.0	234.0	234.5	197.0	3.0	1.0	37.5	0.160
10.5	11.0	236.0	238.0	237.0	197.0	3.2	1.0	40.0	0.169
10.5	12.0	239.0	235.0	237.0	196.0	5.0	1.2	41.0	0.173
10.5	14.8	222.8	222.2	222.5	176.5	5.8	1.2	46.0	0.206
10.5	17.7	241.0	240.6	240.8	186.0	5.8	1.2	54.8	0.228
20.0	22.0	243.0	242.0	242.5	180.0	6.0	1.0	62.5	0.258
20.0	27.2	219.0	218.0	218.5	153.5	6.5	0.7	65.0	0.298
20.0	40.0	235.5	235.5	235.5	147.0	7.5	0.8	88.5	0.376
20.0	72.5	236.0	236.0	236.0	100.0	14.5	0.5	136.0	0.576
20.0	113.0	239.0	243.0	241.0	56.0	19.8	0.7	185.0	0.768
20.0	150.0	248.0	243.0	245.5	29.5	22.5	1.0	216.0	0.880
20.0	185.0	226.0	226.0	226.0	12.0	27.8	2.0	214.0	0.947
20.0	230.0	212.0	213.0	212.5	1.0	27.0	1.5	211.5	0.995
20.0	260.0	233.0	228.0	230.5	−5.0	33.0	2.8	235.5	1.020

apparent secondary electron current. The observations show that practically no primary electrons hit *D*, but nearly all went through the hole into *E*, when *E* was open. The current to *D* was somewhat larger when the target was over the hole in *E*. It was, however, much too small to account for the apparent secondary electron current by a scattering of the primary beam. From Column 5*a*, Table I., we see that there was no current to *D* until a primary velocity of 4 volts (uncorrected)

was reached, after which the ratio of this current to the total secondary current (obtained by taking the ratio of the corresponding values of Column 5*a* and Column 6) increased to about 0.12 at 15 volts primary velocity and then remained more or less constant until 150 volts primary velocity was reached, after which it again increased to about .14 at 260 volts. The magnitude of this current is easily explained if we consider that, of the secondary electrons which leave the target in random directions,[1] a small percentage will be intercepted by *D*, for, although *D* was as close as practicable to the target (something less than one mm.), it subtended an angle sufficient to account for the current which it received.[2] Column 7 was obtained, as previously explained, by dividing the difference between the total current and the current to the target by the total current. Since only the ratio of the currents was desired no attention was paid to their absolute magnitudes. The currents for velocities of the order of a volt and less were easily measured with a D'Arsonval galvanometer of sensibility 10^{-10} amps. per mm. of deflection and much larger currents were obtained for the higher velocities.

The velocity distributions of the primary electrons, obtained as previously explained, are shown in Figs. 4 to 6. The dotted curves in

[1] John T. Tate, PHYS. REV., *17*, p. 394, 1921. C. Davisson and C. H. Kunsman, Science, *54*, p. 522, 1921.

[2] In this case we should expect that the ratio of the current to *D* to the total secondary current would remain constant, independent of the magnitude of the secondary current. The increase of this ratio for the lower primary velocities may be due to the effect of the magnetic field, causing the bulk of the secondary electrons, having sufficiently low velocities, to leave the target in the direction of the field, *i.e.*, along the axis of the tube and thus preventing them from striking *D*. Then, only those secondary electrons which have a velocity sufficient to overcome the effect of the magnetic field would be able to reach *D*. The ratio of this number to the secondary current would obviously increase with the primary velocity until the bulk of the secondary electrons had a velocity great enough to be unaffected by the magnetic field (see velocity distribution curves of secondary electrons below). The increase of current to *D* which begins at the higher velocities may be the result of reflection of the secondary electrons from the inner surface of the cylinder, *C*, back to *D*. Although this reflection is decreased by the presence of gauze on the surface, as previously seen, the velocity distribution curves of secondary electrons, shown below, indicate that there are some high-velocity secondary electrons leaving the target which might produce an appreciable effect due to their reflection from the cylinder *C* back to the diaphragm *D*. Some of these reflected secondary electrons would also strike the target and thus introduce an error since it has been assumed, for the results obtained, that the only electrons hitting the target are primary electrons. This effect, however, would not be noticeable until comparatively large values of the primary velocity had been reached. A rough calculation shows that, for a primary velocity of 30 volts, less than 0.3 per cent. of the incident electrons on the target are secondary electrons from the cylinder *C*. At 100 volts it amounts to less than 1.0 per cent. This error would cause the observed secondary electron curve to lie below the correct curve. In this calculation it was assumed that, due to the effect of the magnetic field, all of the secondary electrons from the target left in directions so as to hit the opposite end of the cylinder *C*, instead of leaving in random directions, and a similar assumption was made for those leaving the end of the cylinder *C*.

Fig. 4 were obtained by plotting the slopes of the original curves as ordinates. The ordinate at any point is then proportional to the number of electrons having a velocity corresponding to the retarding potential.

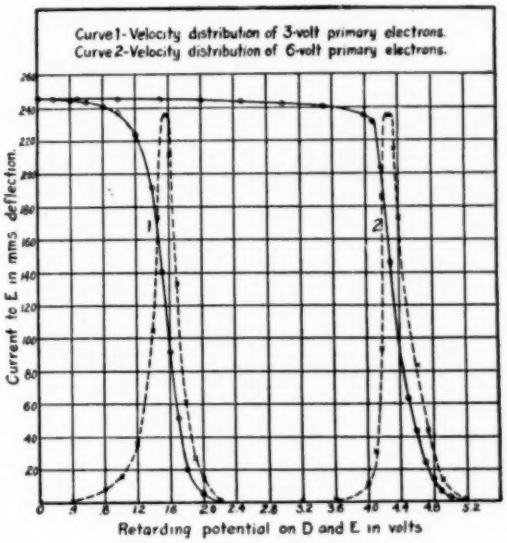
Curve 1- Velocity distribution of 3-volt primary electrons.
Curve 2-Velocity distribution of 6-volt primary electrons.
Current to E in mms deflection.
Retarding potential on D and E in volts
1
2

Fig. 4.

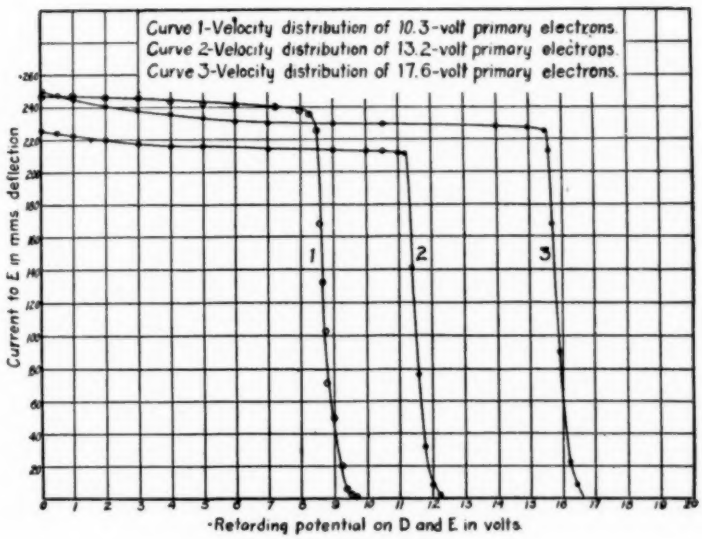
Curve 1-Velocity distribution of 10.3-volt primary electrons.
Curve 2-Velocity distribution of 13.2-volt primary electrons.
Curve 3-Velocity distribution of 17.6-volt primary electrons.
Current to E in mms deflection
Retarding potential on D and E in volts
1
2
3

Fig. 5.

It will be noticed that, although the velocity distribution is probably as good as could be expected, the bulk of the electrons have velocities somewhat lower than that corresponding to the net accelerating voltage

applied. This must be due to a contact potential between the platinum source and the nickel. Because of this difference the velocities as given in Column 2 of Table I. must be corrected. This correction, which varies slightly with the primary velocity, was made before the curves were plotted so that the abscissa of the curves in Figs. 2 and 3 are the corrected velocities. Since the velocity distribution curves show that

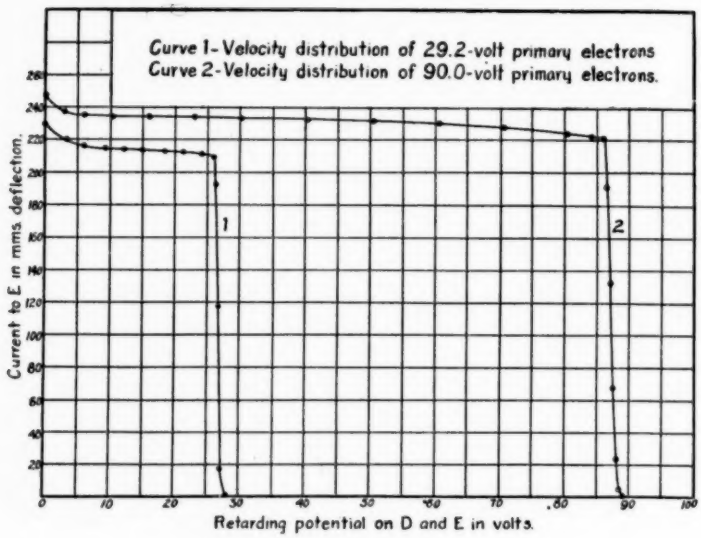
Curve 1- Velocity distribution of 29.2-volt primary electrons
Curve 2- Velocity distribution of 90.0-volt primary electrons.
Current to E in mms. deflection.
Retarding potential on D and E in volts.
1
2

Fig. 6.

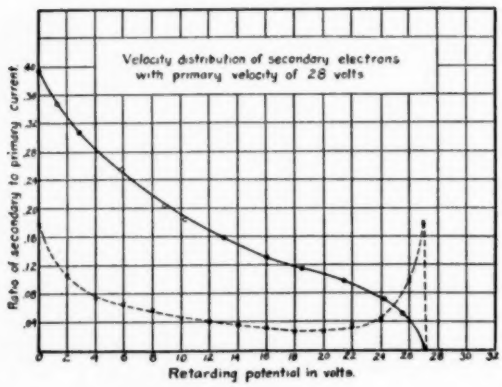
Velocity distribution of secondary electrons with primary velocity of 28 volts
Ratio of secondary to primary current
Retarding potential in volts.

Fig. 7.

the velocities vary over a range of about one volt, the mean value was taken as the correct one. It was found that the velocity distribution of the primary beam depended to a slight extent on the potential of the focus plate, F, the best distribution being obtained with the potential

of F the same as that of the source, S. For this reason S and F were kept at the same potential during all the observations although there were other potentials of F which gave larger primary currents.

The velocity distributions of the secondary electrons, obtained as explained under "procedure," are shown in Figs. 7 to 9. The dotted curves were obtained by plotting the slopes of the other curves as ordi-

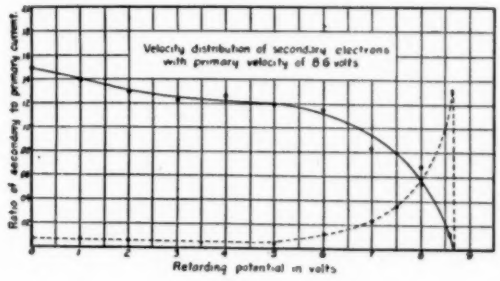
Velocity distribution of secondary electrons with primary velocity of 8.6 volts
Ratio of secondary to primary current
Retarding potential in volts

Fig. 8.

nates. These curves indicate that, when the primary velocity is below about 9 volts, a large percentage of the secondary electrons have velocities nearly equal to the primary velocity and a very small percentage have velocities less than one volt. For higher primary velocities the distribu-

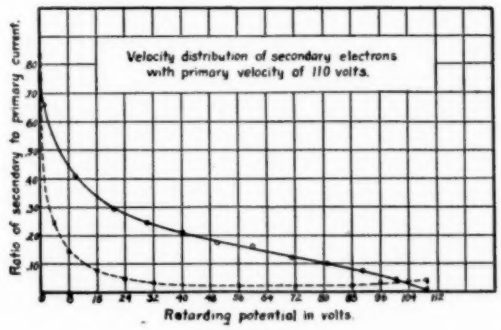
Velocity distribution of secondary electrons with primary velocity of 110 volts.
Ratio of secondary to primary current.
Retarding potential in volts.

Fig. 9.

tion changes, there being many secondary electrons having low velocities. The percentage of secondary electrons having velocities less than 5 volts, say, increases as the primary velocity increases. There are, however, a few per cent. having velocities nearly equal to that of the primary electrons when the primary velocity is as great as 110 volts. As stated in the introduction, Gehrts and von Baeyer found no secondary electrons with velocities greater than 30 volts. The present results, however, agree qualitatively with those of Davisson and Kunsman, who found

some secondary electrons with velocities nearly equal to that of the primary stream.

DISCUSSION OF RESULTS.

In the preceding discussion the use of the words "reflection" and "emission" has purposely been avoided, since there has been some difference of opinion as to whether actual reflection of the primary electrons occurs or whether all of the secondary electrons are produced by ionization at the metal surface. It is certain that emission occurs at high velocities where more electrons leave the surface than strike it, but the question is obviously, whether reflection occurs at all, and if so, at what velocity of impact emission begins. Gehrts and von Baeyer concluded from the nature of their results (see introduction) that reflection occurred only below 11 volts, and that at this velocity an emission began which increased rapidly with the primary velocity. They were, however, unable to interpret the maximum in the secondary electron curve at 5 volts. They considered that their explanation would have seemed more plausible if the number of secondary electrons had decreased steadily with the primary voltage up to 11 volts and then increased, instead of showing a maximum at about 5 volts. The phenomena could then have been considered as due to reflection which was large for small primary velocities but which decreased with increasing velocity to 11 volts, where an emission began and increased rapidly with further increase in primary velocity. The results of the present investigation show that the maximum which they observed at 5 volts was probably due to the effects of absorbed gas, since it disappears with continued heating. Since I. G. Barber obtained no secondary electrons below about 10 volts' primary velocity he concluded that there is no such thing as reflection of electrons and that emission begins at about 10 volts.

The following features of the present investigation show that a new factor enters into the production of secondary electrons from nickel at about 9 volts: (1) the sudden change in slope of the secondary electron curve (Figs. 2 and 3) at about 9 volts; (2) the difference between the velocity distribution of secondary electrons when the primary velocity is 8.6 volts or less, and that when the primary velocity is considerably greater than this. These results lead one to the interpretation that reflection of electrons from nickel occurs for all the primary velocities investigated, the reflection increasing between 0 and 4 volts' primary velocity and then remaining constant to 9 volts, where emission begins and increases with increase of primary velocity to at least 260 volts. That there are some high-velocity electrons present, when the primary velocity is as great as 110 volts, indicates that reflection continues after

emission begins, so that above 9 volts there is both reflection and emission. According to this idea the reflected electrons may be considered to have velocities nearly equal to the primary velocity, *i.e.*, reflection involves a change in direction without much loss in energy, while the emitted electrons have a much smaller velocity.

The above interpretation is further substantiated by a phenomenon which was observed when obtaining the velocity distribution of the primary electrons. It will be noticed, from the velocity distribution curves of the primary electrons, that when the primary velocity was less than 9 volts, all of the primary electrons had velocities nearly equal to that corresponding to the applied voltage. For larger primary velocities, however, there are some low velocity electrons present, as indicated by the rise of the left end of the curves. Also, the number present appears to increase with the primary velocity. This can be explained, with the previous interpretation, as being due to emission of electrons from the edges of the diaphragms which limit the primary beam. Thus, since below 9 volts no emission occurs, there are no low-velocity electrons present in the primary beam.

The accuracy in determining the ionization potential depends, of course, on the velocity distribution of the primary beam. Since the velocity varies over a range of about one volt, the mean velocity was taken as the correct one, as previously mentioned. If, however, the electrons with the highest velocity are sufficient in number to produce an appreciable effect, an error will be introduced equal to the difference between this higher velocity and the mean value. Hence the value obtained may be subject to a small correction.

Since the number of secondary electrons obtained at the low velocities was much greater before the continued heating than after, one must conclude that the reflection of electrons from the atoms of gas on a metal surface is greater than from the metal freed from gas. Since the ionization potential of hydrogen is about 11 volts, the increased secondary current below this velocity, as shown in curve 10, Fig. 3, is probably due to reflection from hydrogen atoms in the surface rather than ionization of them. The fact that curve 10 lies above the limiting curve for all velocities up to 260 volts also shows that more electrons are knocked out of the hydrogen atoms than from the nickel atoms. This is also true for air.

The method used is particularly well adapted to the extension of the investigation to other metals. Targets of the various metals can be inserted and the characteristics of each studied under the same conditions by moving them successively into the path of the primary electron

stream. There is no reason to believe that the secondary electron characteristics should be the same for all metals, and a knowledge of their characteristics in this respect should be of value in connection with the electron theory of metals.

SUMMARY AND CONCLUSION.

1. The secondary electron characteristics of nickel, for low primary velocities, have been investigated in detail. The results are independent of the particular target used and also of the roughness of the surface.

2. Heating the nickel target to bright red heat in vacuo changed the shape of the secondary electron curve (Figs. 2 and 3) and decreased the magnitude of the secondary electron current. Exposing the surface to hydrogen again increased the secondary electron current. The same limiting form of curve was obtained after prolonged heating of the target at bright red heat independently of whether it had previously been exposed to air or hydrogen.

3. A sudden change in slope of the secondary electron curve occurs at about 9 volts' primary velocity, which, together with a difference in the velocity distribution of the secondary electrons for primary velocities below and above 9 volts, suggests that emission or ionization at the surface begins at this voltage and increases with primary velocity, and that between 0 and 9 volts only reflection occurs. The presence of secondary electrons with velocities nearly equal to that of the primary beam for the higher velocities suggests that reflection continues after emission begins.

This problem was suggested by Professor Max Mason and was carried out under the direction of Professor C. E. Mendenhall. The author wishes to express his indebtedness to these gentlemen for many valuable suggestions and criticisms throughout the progress of the work.

THE UNIVERSITY OF WISCONSIN,
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NOTE ON THE SPECIFIC HEAT OF TUNGSTEN.

By Paul F. Gaehr.

Synopsis.

Determination of the Specific Heat of Tungsten.—Various possible sources of error are suggested which must be taken into account if the method recently described by Smith and Bigler is to give accurate and reliable results.

IN the March number of this magazine, page 268, there appeared the abstract of a paper by K. K. Smith and P. W. Bigler on the "Specific Heat of Tungsten." The method there described is a clever one, and deserves a wide application. However, a number of points were overlooked, and the omission of the precautions puts the results in doubt. These points were discussed in part by Worthing (August, 1918) and myself (November, 1918) in this magazine. The points in question will be given below.

1. If the entire filament is used, there is no knowledge about heat conduction from the filament to the leading-in wires. Unless this is known, one cannot say what proportion of the electrical energy supplied goes to the filament.

2. In my article, referred to above, it was shown that unless temperature fluctuations be kept down to about 10° C., then one cannot consider as negligible the second and higher order derivatives in the expansion of $f(T)$ into a Taylor's Series. It would therefore seem wise for Smith and Bigler to modify their method by superimposing a small alternating current on a direct current, and to make sure that the temperature fluctuations from the mean do not exceed about 5° C.

3. Not only does it appear that the specific heat varies linearly with the temperature, which ought to be taken account of in the equations, but also it would seem from both Worthing's work and my own that the specific heat appears to be different for increasing current than for decreasing current. This, no doubt, is due to the fact that the radiation from the filament is not simply expressible as $f(T)$, but that to this function should be added a further term involving the time rate of change of the temperature. An alternating current method takes no account of this.

4. Unless the wave form of the alternating current is a pure sine form, quite free from harmonics, there are serious complications.

5. The presence of the Pyrex cylinder, on which the filament is wound, complicates the case in quite the same way as the leading-in wires, even if not to the same extent.

6. The spiral form of the filament tends to raise the temperature of the filament, so that it is doubtful that Langmuir's data are applicable.

It would seem that the experiment could be repeated, involving the above suggestions (excepting perhaps the third), and further using Worthing's method of potential leads. If, also, the temperature of the filament is determined pyrometrically, the method should then be an excellent one.